

USING NEURAL NETWORK FOR IMPROVING AN EXPLICIT ALGEBRAIC STRESS MODEL IN 2D FLOWS [?]

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- The ML will then be some form of **regression method**.

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- In this work I will use **DNS databases** of channel flow and flat-plate boundary layer flow
- The objective is to improve the Explicit Algebraic Reynolds Stress Model (EARSM)

FLOW EQUATIONS

The momentum equations read

$$\frac{\partial \bar{v}_j \bar{v}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \overline{v'_i v'_j}}{\partial x_j}$$

In EARSM

$$\frac{\partial \bar{v}_j \bar{v}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[(\nu + \nu_{eff}) \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i} \right) \right] - \frac{\partial (\overline{v'_i v'_j})_r}{\partial x_j}$$

where ν_{eff} is the effective viscosity including the EARSM coefficient β_1 , see Eq. 16 in [?]. The last term includes $(\overline{v'_i v'_j})_r$ which is the residual stress tensor in EARSM. The total stress tensor is

$$\overline{v'_i v'_j} = -\nu_{eff} \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i} \right) + (\overline{v'_i v'_j})_r$$

$k - \omega$ TURBULENCE MODE

The Wilcox $k - \omega$ turbulence model reads [?]

$$\frac{\partial \bar{v}_j k}{\partial x_j} = P^k + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] - C_\mu k \omega$$

$$\frac{\partial \bar{v}_j \omega}{\partial x_j} = C_{\omega 1} \frac{\omega}{k} P^k + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\omega} \right) \frac{\partial \omega}{\partial x_j} \right] - C_{\omega 2} \omega^2$$

$$\nu_t = \frac{k}{\omega}$$

The standard coefficients are used, i.e. $C_{\omega 1} = 5/9$, $C_{\omega 2} = 3/40$, $\sigma_k = \sigma_\omega = 2$ and $C_\mu = 0.09$. The production term is computed as

$$P^k = -\overline{v'_i v'_j} \frac{\partial \bar{v}_i}{\partial x_j}$$

and the dissipation in the EARSM reads

$$\varepsilon = C_\mu k \omega$$

The Algebraic Stress Model (ASM) with the LRR pressure-strain models [?] reads

$$\begin{aligned} \left(c_1 - 1 + P^k / \varepsilon \right) a_{ij} = & -\frac{8}{15} \bar{s}_{ij} + \frac{7c_2 + 1}{11} (a_{ik} \bar{\Omega}_{kj} - \bar{\Omega}_{ik} a_{kj}) \\ & - \frac{5 - 9c_2}{11} \left(a_{ik} \bar{s}_{kj} + \bar{s}_{ik} a_{kj} - \frac{2}{3} a_{mn} \bar{s}_{nm} \delta_{ij} \right), \quad a_{ij} = \frac{\overline{v'_i v'_j}}{k} - \frac{2}{3} \delta_{ij} \end{aligned} \quad (1)$$

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Wallin & Johansson [?] and Girimaji [?] derived an explicit form which in 2D which reads

$$a_{ij} = \beta_1 \bar{s}_{ij}^* + \beta_2 \left(\bar{s}_{ik}^* \bar{s}_{kj}^* - \frac{1}{3} \bar{s}_{mn}^* \bar{s}_{nm}^* \delta_{ij} \right) + \beta_4 \bar{s}_{ik}^* \left(\bar{\Omega}_{kj}^* - \bar{\Omega}_{ik}^* \bar{s}_{kj}^* \right) \quad (2)$$

$$\bar{s}_{ij}^* = \frac{k}{\varepsilon} \bar{s}_{ij}, \quad \bar{\Omega}_{ij}^* = \frac{k}{\varepsilon} \bar{\Omega}_{ij}$$

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Note that Eq. 1 is simplified if c_2 is set to 5/9.

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$$\beta_1 = -\frac{A_1 N}{Q}, \quad \beta_2 = 2\frac{A_1 A_2}{Q}, \quad \beta_4 = -\frac{A_1}{Q}, \quad Q = N^2 - 2I_{\Omega} - \frac{2}{3}A_2^2 I_S$$

where N is given by the cubic equation

$$N^3 - A_3 N^2 \left(\left(A_1 A_4 + \frac{2}{3} A_2^2 \right) I_S + 2I_{\Omega} \right) N + 2A_3 \left(\frac{1}{3} A_2^2 I_S + I_{\Omega} \right) = 0 \quad (3)$$

where $I_S = \bar{s}_{mn}^* \bar{s}_{nm}^*$ and $I_{\Omega} = \bar{\Omega}_{mn}^* \bar{\Omega}_{nm}^*$.

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Equation 3 can be solved analytically.

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Instead of using the expression on the previous slide for β_1 , β_2 , β_4 , let's make them functions of something using NN.

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- I will use NN in Python's `pytorch`

EARSM IN FULLY-DEVELOPED CHANNEL FLOW

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$$\beta_1 = \frac{2a_{12}}{\frac{\partial \bar{v}_1^*}{\partial x_2}}, \quad \beta_2 = \frac{6(a_{11} + a_{22})}{\left(\frac{\partial \bar{v}_1^*}{\partial x_2} \right)^2}, \quad \beta_4 = \frac{a_{22} - a_{11}}{\left(\frac{\partial \bar{v}_1^*}{\partial x_2} \right)^2}$$

EARSM, CHANNEL FLOW, $Re_\tau = 2\,000$, $Re_\tau = 5\,200$, $Re_\tau = 10\,000$

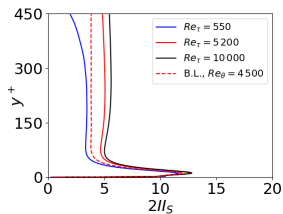
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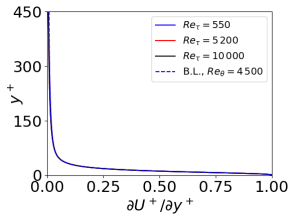
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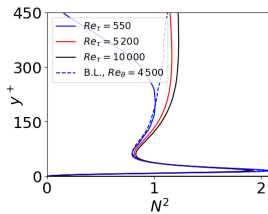
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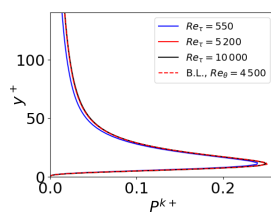
(A) Velocity gradient scaled with k and $\tilde{\varepsilon}$.



(B) Velocity gradient scaled with u_τ and ν .



(C) Ratio of P^k to $\tilde{\varepsilon}$.



(D) Production scaled with u_τ and ν .

FIGURE: DNS data. $\tilde{\varepsilon} = \varepsilon - \nu \partial^2 k / \partial y^2$.

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- The velocity gradient, $\frac{\partial U^+}{\partial y^+}$, could be an option
- But it was found that the NN training process does not converge. Probably because of the large gradients near the wall.
- I scale the two input parameters using `MinMaxScaler()` so that they are in the range $[0, 1]$

NEURAL NETWORK (NN). PYTHON'S `PYTORCH`

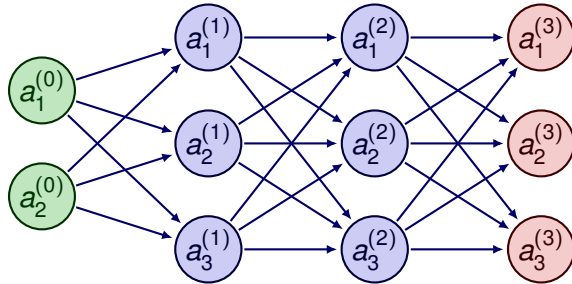
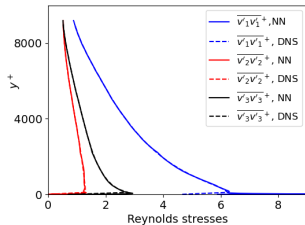


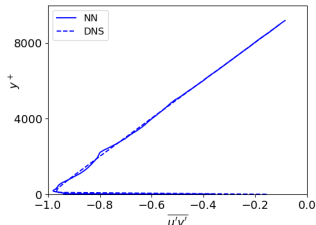
FIGURE: The Neural Network with two input variables, $a_1^{(0)} = y^+$ and $a_2^{(0)} = P^{k+}$ and three output variables, $a_1^{(3)} = \beta_1$, $a_2^{(3)} = \beta_2$ and $a_3^{(3)} = \beta_4$. There are three neurons in this figure; in the simulations I have 50.

PYTHON'S PYTORCH. TRAINING & PREDICTING

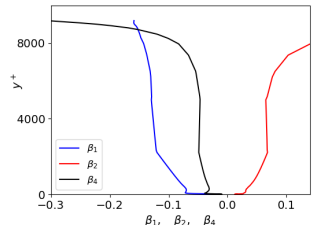
- I train the NN model using DNS data of channel flow at $Re_\tau = 10\,000$.
- I exclude data in the viscous sublayer ($y^+ \leq 5$) and near the center ($y^+ \geq 9\,800$) where $\frac{\partial \bar{v}_1^*}{\partial x_2}$ goes to zero.
- I train on 80% of the data (approx 800 randomly selected DNS data points) and test (predict) on the remaining 20%.



(A) Reynolds normal stresses.



(B) Reynolds shear stress.



(C) Predicted EARSM coefficients.

FIGURE: Predicted with NN. $Re_\tau = 10\,000$. Trained on $Re_\tau = 10\,000$.

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CFD SOLVER

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- Discretization: Hybrid first-order upwind/second-order central differencing
- The discretized equations are solved with **Python sparse matrix solvers**.

STANDARD $k - \omega$ MODEL

$$\begin{aligned}\bar{v}_j \frac{\partial k}{\partial x_j} &= P^k + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] - C_\mu k \omega \\ \bar{v}_j \frac{\partial \omega}{\partial x_j} &= \alpha \frac{\omega}{k} P^k + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\omega} \right) \frac{\partial \omega}{\partial x_j} \right] - \beta \omega^2 \\ \nu_t &= \frac{k}{\omega}\end{aligned}\tag{4}$$

INCLUDING NN PREDICTION IN THE CFD SOLVER

- 1 Load the NN model into **pyCALC-RANS**

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- ① Load the NN model into **pyCALC-RANS**
- ② Solve U , V and PP equations. The Reynolds stresses $\overline{v_1'^2}$, $\overline{v_2'^2}$, $\overline{v_1' v_2'}$ in the U and V equations are taken from previous iteration

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- 3 Compute β_1 , β_2 , β_4 using NN. Use limits from training data.
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- 6 Solve k and ω equations. The Reynolds stresses $\overline{v_1'^2}$, $\overline{v_2'^2}$, $\overline{v_1'v_2'}$ are used in the production terms

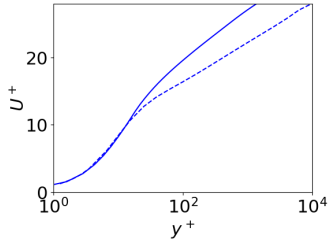
INCLUDING NN PREDICTION IN THE CFD SOLVER

- 1 Load the NN model into **pyCALC-RANS**
- 2 Solve U , V and PP equations. The Reynolds stresses $\overline{v_1'^2}$, $\overline{v_2'^2}$, $\overline{v_1'v_2'}$ in the U and V equations are taken from previous iteration
- 3 Compute β_1 , β_2 , β_4 using NN. Use limits from training data.
- 4 Compute the anisotropic Reynolds stresses (a_{11} , a_{22} , a_{12}) using β_1 , β_2 , β_4
- 5

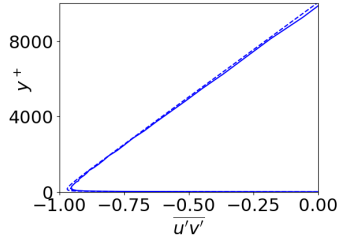
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- 6 Solve k and ω equations. The Reynolds stresses $\overline{v_1'^2}$, $\overline{v_2'^2}$, $\overline{v_1'v_2'}$ are used in the production terms
- 7 End of global iteration. Repeat from Step 2 until convergence (1000s of iterations)

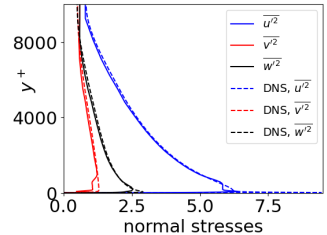
CFD & NN. $Re_\tau = 10\,000$. NN TRAINED ON $Re_\tau = 10\,000$



(A) Velocity.

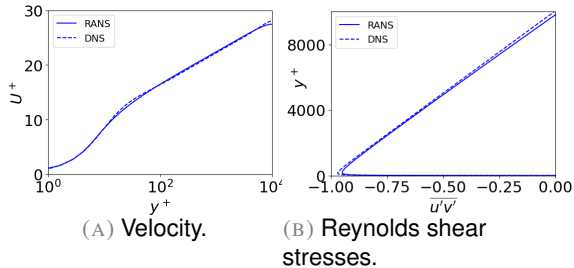


(B) Reynolds shear stresses.



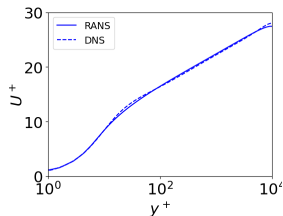
(C) Reynolds normal stresses.

WHY SUCH POOR PREDICTIONS? LET'S LOOK AT $k - \omega$ PREDICTIONS

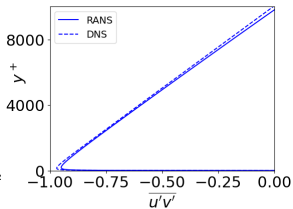


- Velocity and shear stress are (or seems to be) well predicted

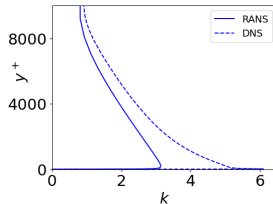
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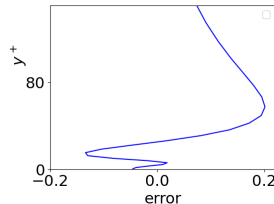
(A) Velocity.



(B) Reynolds shear stresses.



(C) Turb. kinetic energy.



(D) Error
$$\frac{(\partial \bar{u} / \partial y)_{k-\omega} - (\partial \bar{u} / \partial y)_{DNS}}{(\partial \bar{u} / \partial y)_{DNS}}$$

- Velocity and shear stress are (or seems to be) well predicted
- Both k and the velocity gradient are poorly predicted

LET'S TRAIN NN WITH $k - \omega$ DATA AND DNS AT $Re_\tau = 10\,000$

- I use NN with
 - Input: P^k and y^+ from $k - \omega$ prediction
 - Target: β_1, β_2 and β_4 from $\underbrace{\overline{v_1'^2}, \overline{v_2'^2}}_{DNS}$ and $\underbrace{\overline{v_1' v_2'}}_{k-\omega}, k, \varepsilon$
- Note that $k_{DNS} = 0.5 \left(\overline{v_1'^2}_{DNS} + \overline{v_2'^2}_{DNS} + \overline{v_3'^2}_{DNS} \right) \neq k_{k-\omega}$

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- In order to make this approach applicable in 3D:
 - develop a new $k - \omega$ model that accurately predicts k

RESULTS

- I will compare two models.
 - NN EARSM with $k - \omega$ trained in channel flow at $Re_\tau = 10\,000$
 - Standard EARSM with $k - \omega$
- Four flows
 - Channel flow at $Re_\tau = 10\,000$, $Re_\tau = 5\,200$, and $Re_\tau = 2\,000$
 - Flat-plate boundary layer, $Re_\theta = 5\,500$.

CHANNEL FLOW, $Re_\tau = 10\,000$

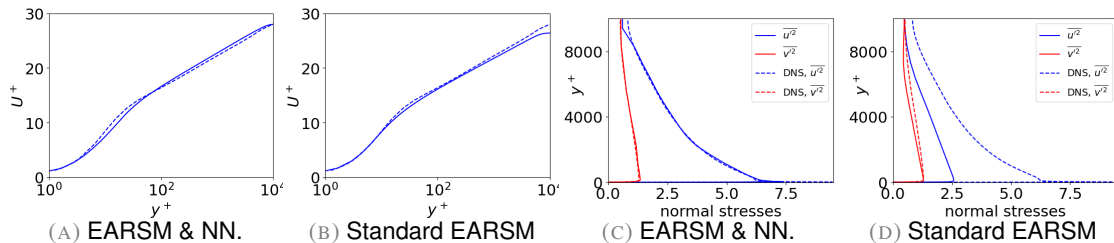


FIGURE: CFD predictions with NN and standard EARSM at $Re_\tau = 10\,000$.

CHANNEL FLOW, $Re_\tau = 5\,200$

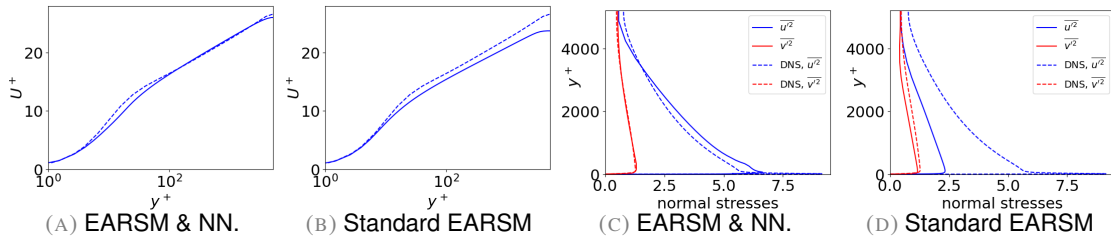


FIGURE: CFD predictions with NN and standard EARSM at $Re_\tau = 5\,200$.

CHANNEL FLOW, $Re_\tau = 2\,000$

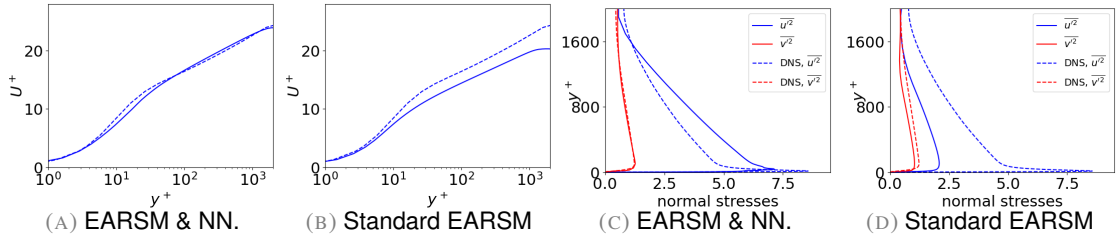


FIGURE: CFD predictions with NN and standard EARSM at $Re_\tau = 2\,000$.

FLAT-PLATE BOUNDARY LAYER, $Re_\theta = 5\,500$.

- Inlet b.c. taken from a pre-cursor $k - \omega$ simulation at $Re_\theta \simeq 2\,500$

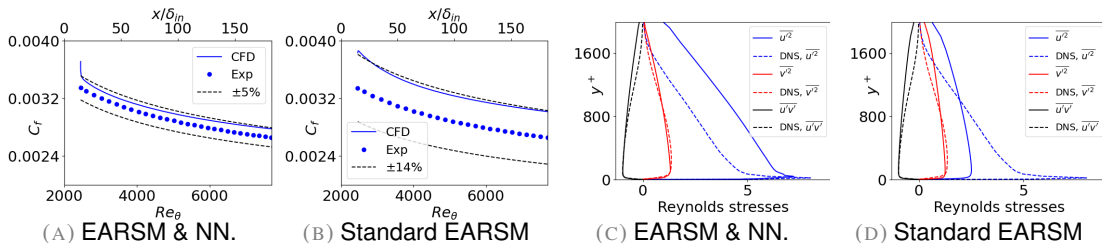
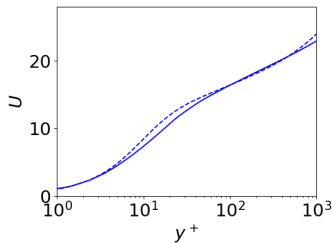
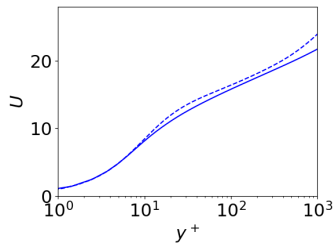


FIGURE: CFD predictions with NN at $Re_\theta = 5\,500$.

FLAT-PLATE BOUNDARY LAYER, $Re_\theta = 5\,500$.



(A) EARSIM & NN.



(B) Standard EARSIM

FIGURE: CFD predictions with NN at $Re_\theta = 5\,500$.

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- Good results are obtained for channel flow at $Re_\tau = 2\,000, 5\,200, 10\,000$ and flat-plate boundary layer (better than the standard EARSM)

- You find the assignment in Section AE.1 in my eBook

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