Using Neural Network for Improving an Explicit Algebraic Stress Model in 2D Flows [2]

Flow-Induced Acoustics Seminar 2024

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- The ML will then be some form of regression method.

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- In this work I will use DNS databases of channel flow and flat-plate boundary layer flow
- The objective is to improve the Explicit Algebraic Reynolds Stress Model (EARSM)

The Algebraic Stress Model (ASM) with the LRR pressure-strain models [5] reads

$$
\begin{aligned}
\left(c_1 - 1 + P^k/\varepsilon\right) a_{ij} &= -\frac{8}{15}\bar{s}_{ij} + \frac{7c_2+1}{11}\left(a_{ik}\bar{\Omega}_{kj} - \bar{\Omega}_{ik}a_{kj}\right) \\
-\frac{5-9c_2}{11}\left(a_{ik}\bar{s}_{kj} + \bar{s}_{ik}a_{kj} - \frac{2}{3}a_{mn}\bar{s}_{nm}\delta_{ij}\right), &\quad a_{ij} = \frac{\overline{v_i'v_j'}}{k} - \frac{2}{3}\delta_{ij}
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Wallin & Johansson [9, 10] and Girimaji [3, 4] derived an explicit form which in 2D which reads

$$a_{ij}=\beta_1\bar{s}^*_{ij}+\beta_2\left(\bar{s}^*_{ik}\bar{s}^*_{kj}-\frac{1}{3}\bar{s}^*_{mn}\bar{s}^*_{nm}\delta_{ij}\right)+\beta_4\bar{s}^*_{ik}\left(\bar{\Omega}^*_{kj}-\bar{\Omega}^*_{ik}\bar{s}^*_{kj}\right)\tag{2}$$

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Note that Eq. 1 is simplified if c_2 is set to $5/9$.

EARSM: ANALYTICAL SOLUTION

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where N is given by the cubic equation

$$N^3 - A_3 N^2 \left(\left(A_1 A_4 + \frac{2}{3}A_2^2\right) II_S + 2II_\Omega\right) N + 2A_3 \left(\frac{1}{3}A_2^2 II_S + II_\Omega\right) = 0 \qquad (3)$$

where $II_S = \bar{s}^*_{mn}\bar{s}^*_{nm}$ and $II_\Omega = \bar{\Omega}^*_{mn}\bar{\Omega}^*_{nm}$.

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Equation 3 can be solved analytically.

Instead of using the expression on the previous slide for β_1, β_2, β_4, let's make them functions of something using NN.

$$a_{ij} = \beta_1 \bar{s}^*_{ij} + \beta_2 \left(\bar{s}^*_{ik} \bar{s}^*_{kj} - \frac{1}{3} \bar{s}^*_{mn} \bar{s}^*_{nm} \delta_{ij} \right) + \beta_4 \bar{s}^*_{ik} \left(\bar{\Omega}^*_{kj} - \bar{\Omega}^*_{ik} \bar{s}^*_{kj} \right), \quad a_{ij} = \frac{\overline{v'_i v'_j}}{k} - \frac{2}{3} \delta_{ij}$$

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$$a_{33} = -\frac{2\beta_2}{12}\left(\frac{\partial \bar{v}_1^*}{\partial x_2}\right)^2, \quad a_{12} = \frac{\beta_1}{2}\frac{\partial \bar{v}_1^*}{\partial x_2}, \quad \frac{\partial \bar{v}_1^*}{\partial x_2} = \frac{k}{\varepsilon}\frac{\partial \bar{v}_1}{\partial x_2}$$

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$$\beta_1 = \frac{2a_{12}}{\frac{\partial \bar{v}_1^*}{\partial x_2}}, \quad \beta_2 = \frac{6(a_{11} + a_{22})}{\left(\frac{\partial \bar{v}_1^*}{\partial x_2}\right)^2}, \quad \beta_4 = \frac{a_{22} - a_{11}}{\left(\frac{\partial \bar{v}_1^*}{\partial x_2}\right)^2}$$

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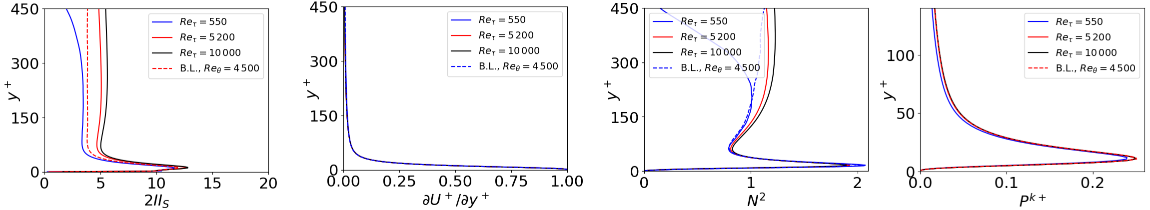

(A) Velocity gradient scaled with k and $\tilde{\varepsilon}$.

(B) Velocity gradient scaled with u_τ and ν.

(C) Ratio of P^k to $\tilde{\varepsilon}$.

(D) Production scaled with u_τ and ν.

FIGURE: DNS data. $\tilde{\varepsilon} = \varepsilon - \nu\partial^2 k/\partial y^2$.

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- I scale the two input parameters using `MinMaxScaler()` so that they are in the range $[0, 1]$

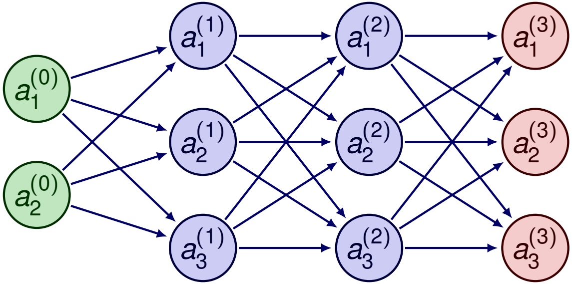
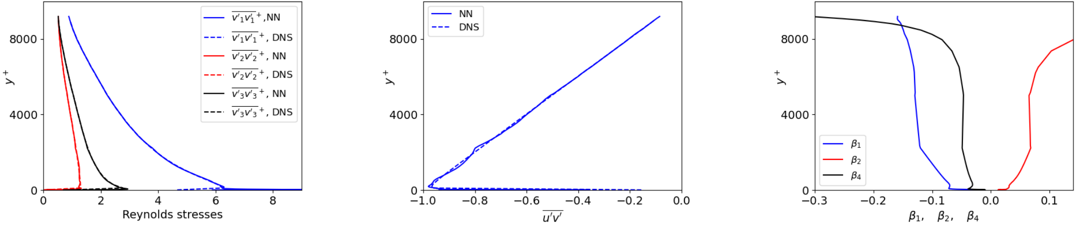


FIGURE: The Neural Network with two input variables, $a_1^{(0)} = y^+$ and $a_2^{(0)} = P^{k+}$ and three output variables, $a_1^{(3)} = \beta_1$, $a_2^{(3)} = \beta_2$ and $a_3^{(3)} = \beta_4$. There are three neurons in this figure; in the simulations I have 50.

- I train the NN model using DNS data of channel flow at $Re_\tau = 10\,000$.
- I exclude data in the viscous sublayer ($y^+ \leq 5$) and near the center ($y^+ \geq 9\,800$) where $\frac{\partial \bar{v}_1^*}{\partial x_2}$ goes to zero.
- I train on 80% of the data (approx 800 randomly selected DNS data points) and test (predict) on the remaining 20%.



(A) Reynolds normal stresses. (B) Reynolds shear stress. (C) Predicted EARSM coefficients.

FIGURE: Predicted with NN. $Re_\tau = 10\,000$. Trained on $Re_\tau = 10\,000$.

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- The discretized equations are solved with Python sparse matrix solvers.

$$
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\bar{v}_j\frac{\partial k}{\partial x_j} &= P^k + \frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] - C_\mu k\omega \\
\bar{v}_j\frac{\partial \omega}{\partial x_j} &= \alpha\frac{\omega}{k}P^k + \frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] - \beta\omega^2 \\
\nu_t &= \frac{k}{\omega}
\end{aligned}
\tag{4}
$$

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INCLUDING NN PREDICTION IN THE CFD SOLVER

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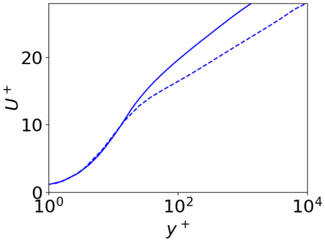
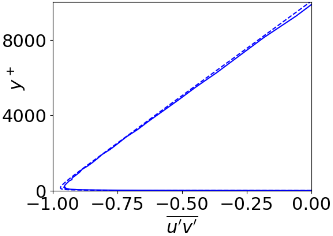
7. End of global iteration. Repeat from Step 2 until convergence (1000s of iterations)

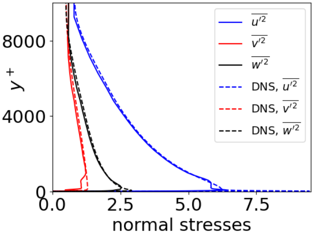
CFD & NN. $Re_\tau = 10\,000$. NN TRAINED ON $Re_\tau = 10\,000$



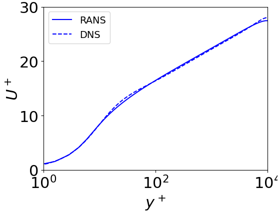
(A) Velocity.



(B) Reynolds shear stresses.



(C) Reynolds normal stresses.



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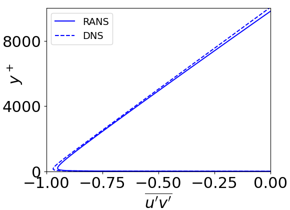

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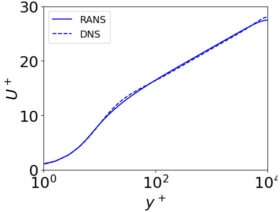
- Velocity and shear stress are (or seems to be) well predicted



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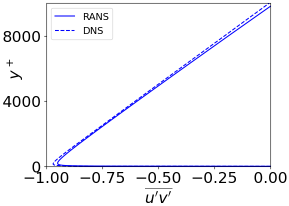

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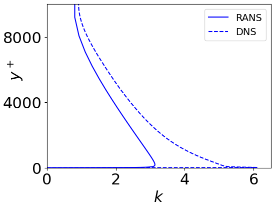
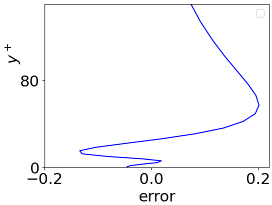


(C) Turb. kinetic energy.



(D) Error $\frac{(\partial \bar{u}/\partial y)_{k-\omega} - (\partial \bar{u}/\partial y)_{DNS}}{(\partial \bar{u}/\partial y)_{DNS}}$

- Velocity and shear stress are (or seems to be) well predicted
- Both k and the velocity gradient are poorly predicted

Let's train NN with $k-\omega$ data and DNS at $Re_\tau = 10\,000$

- I use NN with
  - Input: P^k and y^+ from $k-\omega$ prediction
  - Target: β_1, β_2 and β_4 from $\underbrace{\overline{v_1'^2}, \overline{v_2'^2}}_{DNS}$ and $\underbrace{\overline{v_1' v_2'}, k, \varepsilon}_{k-\omega}$
- Note that $k_{DNS} = 0.5\left(\overline{v_1'^2}_{DNS} + \overline{v_2'^2}_{DNS} + \overline{v_3'^2}_{DNS}\right) \neq k_{k-\omega}$

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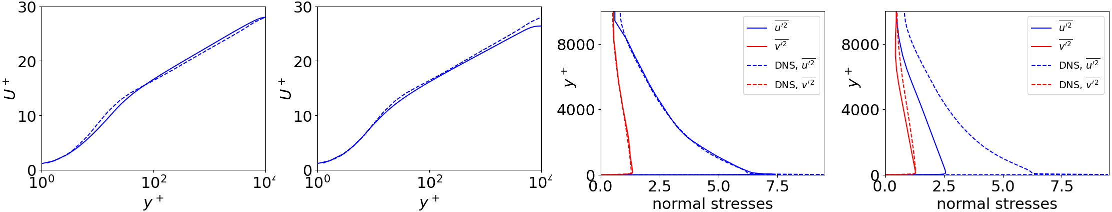
  - develop a new $k-\omega$ model that accurately predicts k

- I will compare two models.
  - NN EARSM with $k-\omega$ trained in channel flow at $Re_\tau = 10\,000$
  - Standard EARSM with $k-\omega$
- Four flows
  - Channel flow at $Re_\tau = 10\,000$, $Re_\tau = 5\,200$, and $Re_\tau = 2\,000$
  - Flat-plate boundary layer, $Re_\theta = 5\,500$.



(A) EARSM & NN. (B) Standard EARSM (C) EARSM & NN. (D) Standard EARSM

Figure: CFD predictions with NN and standard EARSM at $Re_\tau = 10\,000$.

CHANNEL FLOW, $Re_\tau = 5\,200$

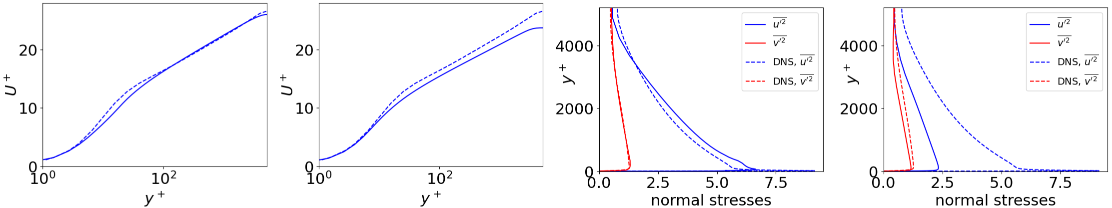


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FIGURE: CFD predictions with NN and standard EARSM at $Re_\tau = 5\,200$.

CHANNEL FLOW, $Re_\tau = 2\,000$

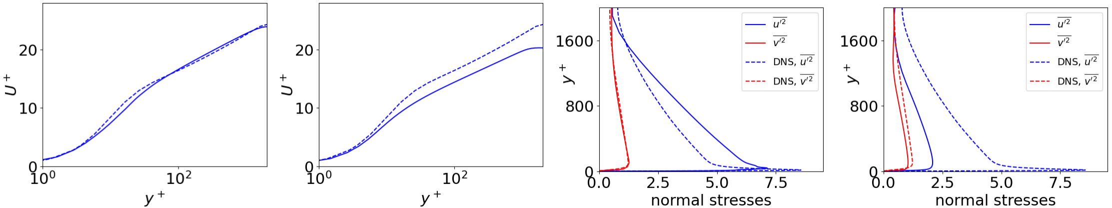

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FIGURE: CFD predictions with NN and standard EARSM at $Re_\tau = 2\,000$.

FLAT-PLATE BOUNDARY LAYER, $Re_\theta = 5\,500$.

- Inlet b.c. taken from a pre-cursor $k-\omega$ simulation at $Re_\theta \simeq 2\,500$

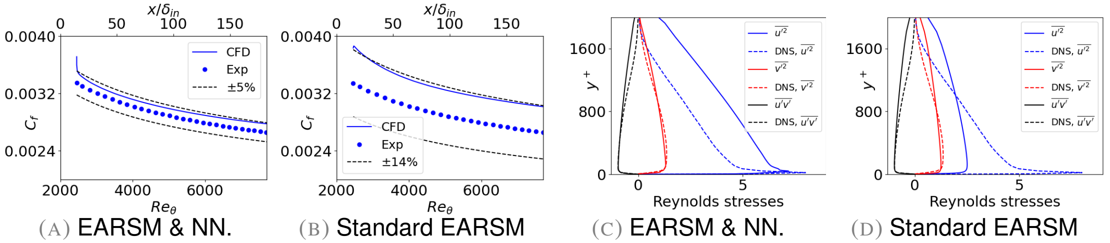

FIGURE: CFD predictions with NN at $Re_\theta = 5\,500$.

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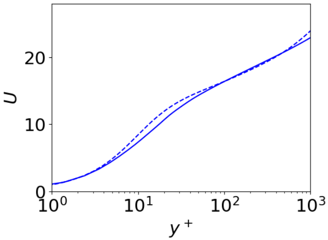
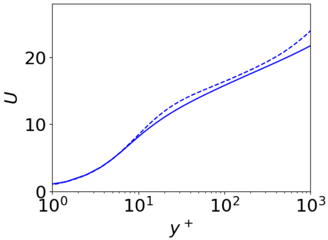


(A) EARSM & NN.



(B) Standard EARSM

FIGURE: CFD predictions with NN at $Re_\theta = 5\,500$.

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