

USING PHYSICAL INFORMED NEURAL NETWORK (PINN) TO IMPROVE A $k - \omega$ TURBULENCE MODEL [2]

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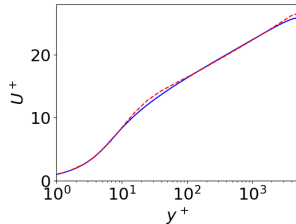
THE $k - \omega$ TURBULENCE MODEL

The Wilcox $k - \omega$ turbulence model reads [4]

$$\begin{aligned}\frac{\partial \bar{v}_i}{\partial x_i} &= 0 \\ \frac{\partial \bar{v}_i}{\partial t} + \frac{\partial \bar{v}_i \bar{v}_j}{\partial x_j} &= -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left[(\nu + \nu_t) \frac{\partial \bar{v}_i}{\partial x_j} \right] \\ \frac{\partial \bar{v}_j k}{\partial x_j} &= P^k + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] - C_\mu k \omega \\ \frac{\partial \bar{v}_j \omega}{\partial x_j} &= C_{\omega 1} \frac{\omega}{k} P^k + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\omega} \right) \frac{\partial \omega}{\partial x_j} \right] - C_{\omega 2} \omega^2 \\ P^k &= \nu_t \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i} \right) \frac{\partial \bar{v}_i}{\partial x_j}, \quad \nu_t = \frac{k}{\omega}\end{aligned}\tag{1}$$

The standard coefficients are used, i.e. $C_{\omega 1} = 5/9$, $C_{\omega 2} = 3/40$, $\sigma_k = \sigma_\omega = 2$ and $C_\mu = 0.09$.

FULLY-DEVELOPED CHANNEL FLOW, $k - \omega$ MODEL, $Re_\tau = 5\,200$

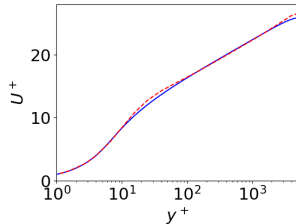


(A) Mean velocity.

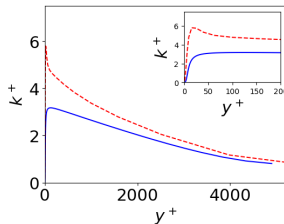
FIGURE: Fully-developed channel flow. Solid lines: $k - \omega$; dashed lines: DNS [3].

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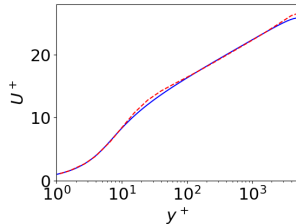


(B) Turb. kinetic energy.

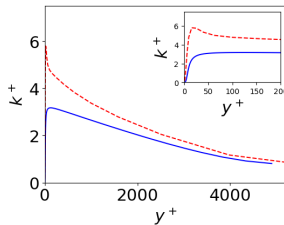
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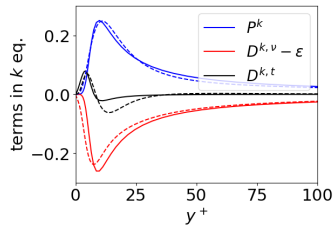
FULLY-DEVELOPED CHANNEL FLOW, $k - \omega$ MODEL, $Re_\tau = 5200$



(A) Mean velocity.



(B) Turb. kinetic energy.



(C) Terms in k equation.

FIGURE: Fully-developed channel flow. Solid lines: $k - \omega$; dashed lines: DNS [3].

- The mean flow, shear stress (and hence the turbulent viscosity, ν_t) agree well
- But not the turbulent, kinetic energy
- It seems to be because the diffusion of k is poorly predicted

FIND A NEW $\nu_{t,k}$

- Our ordinary differential equation reads in fully-developed channel flow

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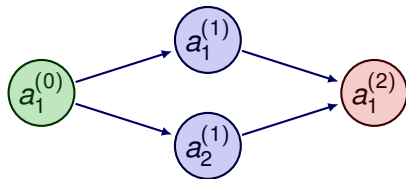
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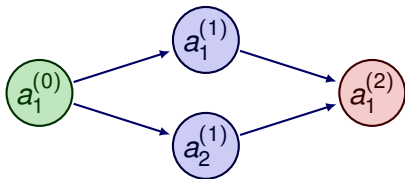
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- $\nu_{t,k} = \nu_{t,NN}$ in Eq. 2, will be predicted by PINN while minimizing the error Q^2 .

NEURAL NETWORK (NN). PYTHON'S `PYTORCH`. CRASH COURSE



- I create a NN that finds a damping function, $Y \equiv f$, as a function of input $X \equiv y^+$
- 1 input ($X = a_1^{(0)}$), 1 hidden layer with 2 neurons ($a_1^{(1)}, a_2^{(1)}$) and 1 output ($Y = a_1^{(2)}$)

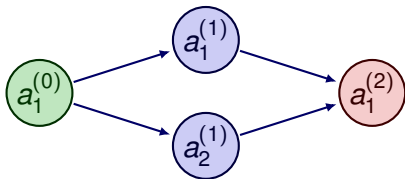
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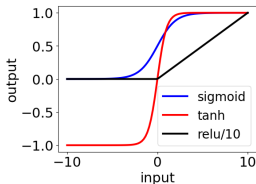
```
class NN(nn.Module):  
    def super__init__(self):  
        self.layer_1=nn.Linear(1, 2) # Connection 0-1  
        self.layer_2=nn.Linear(2, 1) # Connection 1-2  
    def forward(self, x):  
        y = torch.nn.functional.sigmoid(self.layer_1(x)) #  $a_1^{(1)}, a_2^{(1)}$ , hidden-layer  
        output = torch.nn.functional.sigmoid(self.layer_2(y)) #  $a_1^{(2)}$ , output-layer
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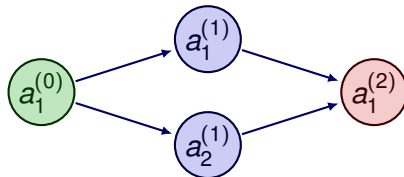


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NEURAL NETWORK (NN). FORWARD



Activation 1:
$$a_1^{(1)} = s_1^{(1)} \left(w_1^{(0)} a_1^{(0)} + b_1^{(0)} \right)$$

Activation 2:
$$a_2^{(1)} = s_2^{(1)} \left(w_2^{(0)} a_1^{(0)} + b_2^{(0)} \right)$$

Output:
$$a_1^{(2)} = s_1^{(2)} \left(w_1^{(1)} a_1^{(1)} + b_1^{(1)} + w_2^{(1)} a_2^{(1)} + b_2^{(1)} \right) \equiv Y$$

- s is an activation function (linear, sigmoid, tanh, ...)

NEURAL NETWORK (NN). BACKWARD

The Python code for the simple NN model is given in the listing below

```
# initiate the NN model
model = NN()
# define input, X
X=np.zeros(nj,1)
X[:,0] = scaler_yplus.fit_transform(yplus)[: ,0]
# define output, Y (f is known)
Y = f
# Training loop
for epoch in range(max_no_epoch):
    # Compute prediction and loss, L
    o = model(X) #prediction
    L = loss_fn(o, Y) # L=|o-Y|_2
    L.backward()
```

- `loss.backward()` computes $dL/dw_1, dL/db_1, dL/ds_1, \dots$
- They are used to get new improved w_1, b_1, \dots

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```
def ODE(y, nut):
    nut_y = torch.autograd(nut, y, torch.ones(x.size()[0], 1), create_graph=True)[0]
    # Differential equation loss
    ODE_loss = (nu+nut)*k_yy + k_y*nut_y + Pk - eps
    ODE_loss = torch.sum(ODE_loss ** 2)
    # b.c. loss
    BC_loss = (nut[0] - nut_0) ** 2
    return ODE_loss, BC_loss
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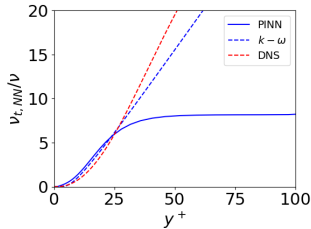
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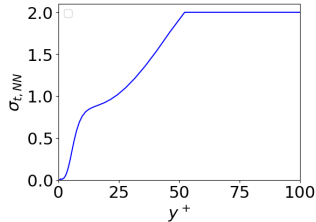
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- `nut` is the unknown
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- There are two losses, `ODE_loss` and `BC_loss`

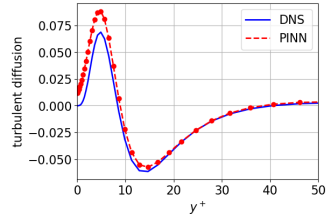
SOLVING EQ. 2 WITH PINN.



(A) Turbulent viscosity.



(B) Prandtl number.



(C) Turbulent diffusion.

FIGURE: k equation.

- Fully-developed flow in half a channel at $Re_\tau = 5\,200$.
- $\sigma_{t,NN} = \nu_t / \nu_{t,NN}$ (ν_t is the turbulent viscosity predicted by the Wilcox $k - \omega$ model)
- $\sigma_{t,NN}$ is limited to 2 (same as σ_k in the Wilcox $k - \omega$ model)

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- The discretized equations are solved with **Python sparse matrix solvers**.

CFD, $Re_\tau = 5\,200$

The equation below is solved is using **pyCALC-RANS**

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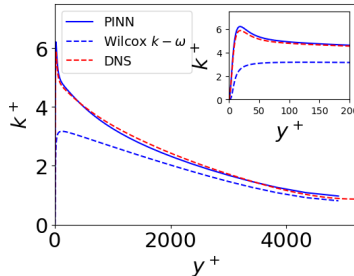


FIGURE: Turbulent kinetic energy.

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- Finally, the ω equation in the new $k - \omega$ model must predict $\omega = \omega_{DNS}$

$$\frac{d}{dy} \left(\frac{\nu_t}{\sigma_\omega} \frac{d\omega_{DNS}}{dy} \right) + C_{\omega 1} \frac{\omega_{DNS}}{k_{DNS}} P_{DNS}^k - C_{\omega 2} \omega_{DNS}^2 = 0$$

PLOT C_k AND $C_{\omega 2}$

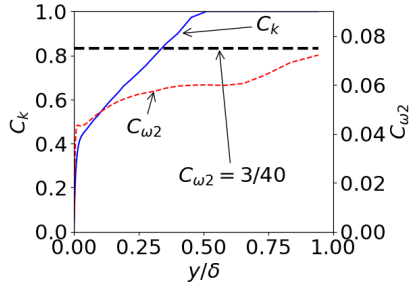
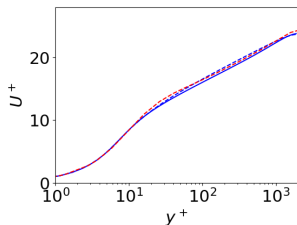
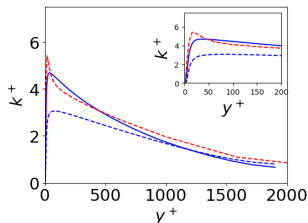


FIGURE: C_k and $C_{\omega 2}$ vs. y/δ .

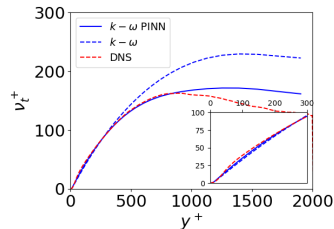
RESULTS. CHANNEL FLOW. $Re_\tau = 2\,000$



(A) Velocity.



(B) Turb. kinetic energy.



(C) Turbulent viscosity.

FIGURE: Fully-developed channel flow. $Re_\tau = 2\,000$.

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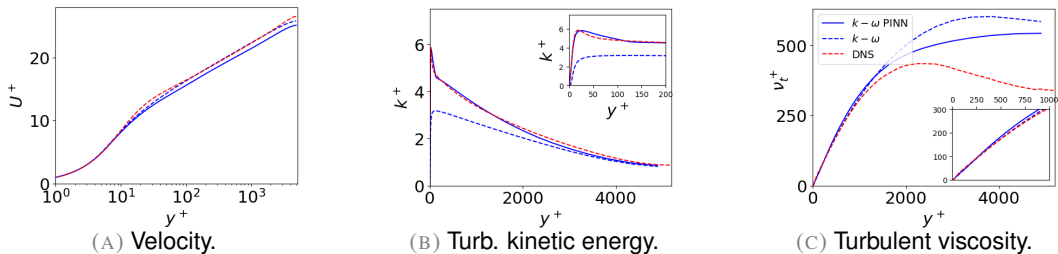
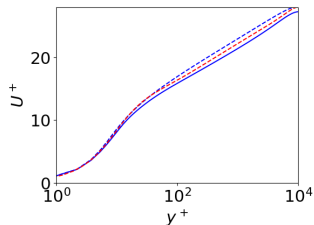
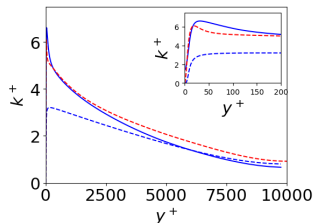


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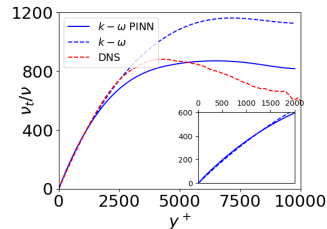
RESULTS. CHANNEL FLOW. $Re_\tau = 10\,000$



(A) Velocity.



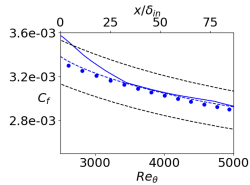
(B) Turb. kinetic energy.



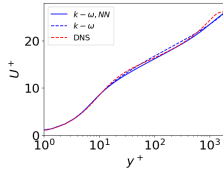
(C) Turbulent viscosity.

FIGURE: Fully-developed channel flow. $Re_\tau = 10\,000$.

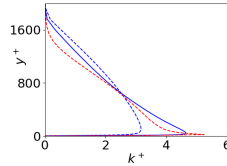
RESULTS. FLAT-PLATE BOUNDARY LAYER



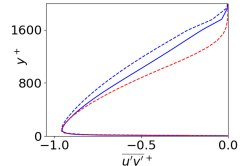
(A) Skin friction.



(B) Velocity.



(C) Turb. kinetic energy.



(D) Turbulent shear stress.

FIGURE: Flat-plate boundary layer. Profiles at $Re_\theta = 4\,500$.

- Inlet profiles from a pre-cursor RANS at $Re_\theta = 2\,500$

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- Finally, I found a good combination input parameters: $\overline{u'v'}/u_\tau^2$ and $\nu_t/(yu_\tau)$ (not shown)

CONCLUDING REMARKS

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- You can download the ETMM15 paper, pyCALC-RANS and PINN scripts [here](#) or Google [pyCALC-RANS PINN](#)

- **Neural Network** and **PINN** in Python.
 - Good YouTube lectures: "3Blue1Brown: But what is a neural network"; "3Blue1Brown: gradient descent, how neural networks learn"; "3Blue1Brown: backpropagation, intuitively"; "3Blue1Brown: backpropagation, calculus"; "Sebastian Lagae: how to create a neural network".

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Download the code [here](#), 2021.
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