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COMPARISON OF DIFFERENT SUBGRID TURBULENCE MODELS AND BOUNDARY CONDITIONS FOR LARGE-EDDY-SIMULATIONS OF ROOM AIR FLOWS.

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ABSTRACT

The calculation results applying two equation turbulence models for the time average Navier-Stokes equations suffer sometimes from large difference to measurements of complex room air flows. Using less modelling assumptions the Large-Eddy-Simulation (LES) becomes practical to calculate room air flows because of more powerful CFD codes and computers. This paper focus on different LES-Models and its boundary conditions.

KEYWORDS

Large-Eddy-Simulation, subgrid models, isothermal room air flows, boundary conditions.

INTRODUCTION

Room air flows are complex flows with low Reynolds numbers. Different flow regimes as attached jets, jets and free convection flows exist in ventilated room. This combination of different flow types causes usually problems using time averaged Navier-Stokes equations with standard two equation turbulence models. The main reasons for this difficulties are non isotrope effects as thermal stratification and the relaminarization from turbulent to laminar flow fields. LES resolves most of the turbulent motion and needs only model assumptions for the less energy containing small scale turbulence.

The first LES model was proposed by Smagorinsky (1963). This model uses local mixing length formulation for the subgridscale stress by applying the local grid space as length scale. Some limitation of the Smagorinsky model were overcame by the first dynamic subgrid model proposed by Germano et al (1991). But this model has numerical stability problems and the remedy is to average the dynamic part of the model in some homogeneous flow direction(s) or to introduce some artificial clipping. Thus, this type of models does not seem to be applicable to real three-dimensional flows as room air flows without introducing ad hoc user modifications.

In the present study two different one-equation subgrid model are compared which eliminate the need of this type of user modifications. One model was presented by Davidson (1997) (DAV) and the idea of this model is to include all local dynamic information into the source terms of the transport equation of K_{SGS}. The second model was presented by W. Kim and S. Menon (1996) (KIM). This model is fully localised and uses no averaging in space or time for the dynamic coefficient C_τ. Similarity of the resolved and non resolved stress tensor is used to calculate the dynamic coefficient. This assumption leads to a more stable formulation for C_τ and gives the possibility to use the local value of C_τ in the momentum equation. The models will be tested for recirculating air flow in a ventilated room.

FILTERING PROCEDURE

LES uses a space averaging technique instead of the usually performed time averaging of the Navier-Stokes equations. All flow quantities have to be averaged in space by a local filter function G. Here, the a simple top-hat filter function G will be used.

$$u_i = \overline{u}_i + u_i' \ ; \ \overline{u} = \int_\Omega u\left(\vec{x}^*\right) G\left(\vec{x}-\vec{x}^*\right) d\vec{x}^* \ ; \ G\left(\vec{x},\vec{x}^*\right) = \begin{cases} 1/\Delta \ \forall \left|\vec{x}-\vec{x}^*\right| \le \dfrac{\Delta}{2} \\ 0 \end{cases} \qquad (1)$$

Applying this filter function G to the continuity and momentum equations gives the following equations:

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \ ; \ \frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \nu \frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j} \ ; \ \tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j \qquad (2)$$

Similar to the time averaging procedure we get an unknown turbulent stress tensor τ_{ij} that relates the unknown correlation of the non filtered quantities to the known filtered ones. This tensor is modelled with a Boussinesq approximation assuming a proportionality of the stress tensor and the local gradient of the filtered quantities.

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_K \ =_K 2\nu_t \left[\overline{S}_{ij} - \frac{1}{3}\delta_{ij}\overline{S}_K \ \right] ;_K \overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) \qquad (3)$$

The isotropic part of the stress tensor is combined with the pressure term leading to the following formulation of the momentum equations for LES:

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{P}}{\partial x_i} + \nu \frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} + \frac{\partial}{\partial x_j}\left[\nu_t \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) \right] \qquad (4)$$

LARGE EDDY SUBGRID MODELS

Smagorinsky model

The first LES subgridscale model uses a mixing length hypotheses based on the local grid dimensions. The turbulent viscosity is calculated as a product of this length scale and a turbulent velocity scale based on the filtered-field deformation tensor $\overline{S}_{ij}$.

$$l = \overline{\Delta} = \left(\Delta x \ \Delta y \ \Delta z\right)^{1/3} \ ; \ \nu_t = \left(C_S \overline{\Delta}\right)^2 \left|\overline{S}\right| \ ; \ \left|\overline{S}\right| = \left(2 \ \overline{S}_{ij}\overline{S}_{ij}\right)^{1/2} \qquad (5)$$

The constant C_S is set to 0.18 but this value does not give reasonable results for all flow types. This model is only able to predict dissipation of resolved turbulent energy. Backscattering of turbulent energy from small scales to larger scales is not possible, because the dissipation at subgrid level is always negativ.

$$\varepsilon_{SGS} = -\left(C_S\,\overline{\Delta}\right)^2 \left|\overline{S}\right|^3 \leq 0 \tag{6}$$

Germano model

To calculate dynamically the value of C_S Germano (1991) applied a second test filter to the filtered Navier-Stokes equation using a larger filter length $\widehat{\Delta}$, here $\widehat{\Delta} = 2\,\overline{\Delta}$.

$$\frac{\partial \hat{\overline{u}}_i}{\partial t} + \frac{\partial \hat{\overline{u}}_i \hat{\overline{u}}_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \hat{\overline{P}}}{\partial x_i} + \nu\left(\frac{\partial \hat{\overline{u}}_i}{\partial x_j \partial x_j}\right) - \frac{\partial T_{ij}}{\partial x_j}\ ;\ \ T_{ij} = \widehat{\overline{u_i u_j}} - \hat{\overline{u}}_i \hat{\overline{u}}_j \tag{7}$$

The resolved stress between the two different filter level L_{ij} is determined by the resolved velocity field and expresses the difference between the test and the grid filtered stress tensor.

$$L_{ij} = \widehat{\overline{u}_i \overline{u}_j} - \hat{\overline{u}}_i \hat{\overline{u}}_j = T_{ij} - \hat{\tau}_{ij} \tag{8}$$

Using the Boussinesq approximation for both filter levels gives

$$\tau_{ij} - \frac{\delta_{ij}}{3}\tau_K = -2\,C_S\,\overline{\Delta}^2 \left|\overline{S}\right| \overline{S}_{ij} = -2\,C_S\,\beta_{ij}\ ;\ \ T_{ij} - \frac{\delta_{ij}}{3}T_K = -2\,C_S\,\widehat{\Delta}^2 \left|\hat{\overline{S}}\right| \hat{\overline{S}}_{ij} = -2\,C_S\,\alpha_{ij} \tag{9}$$

and the constant C_S can be estimated using the non isotropic part of L_{ij}.

$$L^{\alpha}_{ij} = -2C_S\,\alpha_{ij} + 2\widehat{C_S\,\beta_{ij}} \approx -2C_S\,\alpha_{ij} + 2C_S\,\widehat{\beta}_{ij}\ ;\ \ C_S(\vec{x},t) = -\frac{1}{2}\frac{L^{\alpha}_{ij}\left(\alpha_{ij} - \widehat{\beta}_{ij}\right)}{\left(\alpha_{mn} - \widehat{\beta}_{mn}\right)\left(\alpha_{mn} - \widehat{\beta}_{mn}\right)} \tag{10}$$

No predefined constant is necessary to close the equation set but the model suffers from its numerical stability problems and the remedy is to average the dynamic part of the model in some homogeneous flow direction(s) or to introduce some artificial clipping.

One equation model (DAV)

The idea of this one equation model is to include all local dynamic information into the source terms of the transport equation for the kinetic energy of the subgridscales K_{SGS}.

$$K_{SGS} = \frac{1}{2}\tau_{ii} = \frac{1}{2}\left(\overline{u_i u_i} - \overline{u}_i \overline{u}_i\right)\ ;\ \ \nu_t = C\,\overline{\Delta}\,K_{SGS}^{1/2} \tag{11}$$

Additionally a kinetic energy K for the second filter level is defined as:

$$K = \frac{1}{2}T_{ii} = \frac{1}{2}\left(\widehat{\overline{u_i u_i}} - \hat{\overline{u}}_i \hat{\overline{u}}_i\right) \tag{12}$$

Now, assuming similar transport of this turbulent energies on both level

$$C_{K_{SGS}} - D_{K_{SGS}} = P_{K_{SGS}} - \varepsilon_{K_{SGS}}\ ;\ \ C_K - D_K = P_K - \varepsilon_K \tag{13}$$

and the same formulation of the dissipation

$$\varepsilon_{K_{SGS}} = C_\varepsilon \frac{K_{SGG}^{3/2}}{\overline{\Delta}}\ ;\ \ \varepsilon_K = C_\varepsilon \frac{K^{3/2}}{\widehat{\Delta}} \tag{14}$$

yields to the following equation by using the second filter for the RHS of the K_{SGS} transport equation. This term should be comparable to the non filtered RHS of the K transport equation.

$$\widehat{P_{K_{SGS}}} - \widehat{C_\varepsilon \frac{K_{SGS}^{3/2}}{\overline{\Delta}}} = P_K - C_\varepsilon \frac{K^{3/2}}{\widehat{\Delta}} \tag{15}$$

Based on the old values for C_ε a new value is defined locally for every time step.

$$C_\varepsilon^{n+1} = \left(P_K - \widehat{P_{K_{SGS}}} + \widehat{C_\varepsilon^n \frac{K_{SGS}}{\overline{\Delta}}}\right)\frac{\widehat{\Delta}}{K^{3/2}} \tag{16}$$

This local value for C_ε is used in the dissipation formulation of the transport equation for $K_{SGS.}$

$$\frac{\partial K_{SGS}}{\partial t}+\frac{\partial \overline{u}_j K_{SGS}}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\langle C\rangle_{xyz}\,\overline{\Delta} K_{SGS}^{1/2}\,\frac{\partial K_{SGS}}{\partial x_j}\right)+2\nu_t\overline{S}_{ij}\overline{S}_{ij}-C_\varepsilon\,\frac{K_{SGS}^{3/2}}{\overline{\Delta}} \tag{17}$$

In diffusive term in the K_{SGS} equation above as well as in the momentum equations an average value of C is used which is computed with the requirement that the production in the whole domain remains the same.

$$\left\langle 2C\overline{\Delta}K_{SGS}^{1/2}\ \overline{S}_{ij}\overline{S}_{ij}\right\rangle_{xyz}=2\langle C\rangle_{xyz}\ \left\langle\overline{\Delta}K_{SGS}^{1/2}\ \overline{S}_{ij}\overline{S}_{ij}\right\rangle_{xyz} \tag{18}$$

One equation model (KIM)

The one equation model of Menon and Kim is based on the following similarity assumption of different scales $\tau_{ij}=C_k L_{ij}$. They define a kinetic energy K_{test} of a test level that includes all energy between two length scales ($\overline{\Delta}<l<\widehat{\Delta}$).

$$K_{test}=\frac{1}{2}\left(\widehat{\overline{u}_k\overline{u}_k}-\widehat{\overline{u}}_k\widehat{\overline{u}}_k\right) \tag{19}$$

Assuming a similar representation of this two levels we get

$$\tau_{ij}=-2C_\tau\,\overline{\Delta}K_{SGS}^{1/2}\overline{S}_{ij}+\frac{\delta_{ij}}{3}\tau_{kk}\ ;\ \ L_{ij}=-2C_\tau\,\widehat{\Delta}K_{test}^{1/2}\widehat{\overline{S}}_{ij}+\frac{\delta_{ij}}{3}L_{kk} \tag{20}$$

and one can calculate the value of directly as shown.

$$C_\tau=\frac{1}{2}\frac{L_{ij}\sigma_{ij}}{\sigma_{ij}\sigma_{ij}}\ ;\ \ \sigma_{ij}=-\widehat{\Delta}K_{test}^{1/2}\widehat{\overline{S}}_{ij} \tag{21}$$

Dissipation of energy at the test level takes place between the two length scales:

$$\varepsilon_{test}=(\nu+\nu_t)\left[\widehat{\frac{\partial\overline{u}_i}{\partial x_j}\frac{\partial\overline{u}_i}{\partial x_j}}-\frac{\partial\widehat{\overline{u}}_i}{\partial x_j}\frac{\partial\widehat{\overline{u}}_i}{\partial x_j}\right]\ ;\ \ \varepsilon_{test}=C_\varepsilon\,\frac{K_{test}^{3/2}}{\widehat{\Delta}}\Rightarrow C_\varepsilon\Rightarrow\varepsilon_{SGS}=C_\varepsilon\,\frac{K_{test}^{3/2}}{\overline{\Delta}} \tag{22}$$

This model gives a transport equation for K_{SGS} that is able to use the local value of C_τ in the diffusion and C_ε in the dissipation term. To prevent negative diffusion the total diffusion (laminar and turbulent) is limited to positive values.

$$\frac{\partial K_{SGS}}{\partial t}+\frac{\partial \overline{u}_j K_{SGS}}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\underbrace{C_\tau\,\overline{\Delta}K_{SGS}^{1/2}}_{>\ -\nu}+\nu\right)\frac{\partial K_{SGS}}{\partial x_j}\right]+2\nu_t\overline{S}_{ij}\overline{S}_{ij}-C_\varepsilon\,\frac{K_{SGS}^{3/2}}{\overline{\Delta}} \tag{23}$$

BOUNDARY CONDITIONS

A common inlet boundary condition for LES is white noise. Using a random number for every time step to change the inlet boundary condition as shown in the equation below gives the desired turbulence level.

$$u_{in}=\overline{u}_{in}+\sqrt{12}\,t_{in}\overline{u}_{in}\xi \qquad\qquad -0{,}5<\xi<0{,}5 \tag{24}$$

But this kind of artificial turbulence has no length scale because this fluctuations have no correlation in space or time. The dissipation is very large and thus, this fluctuation will vanish fast.

$$l=\int f(r)\,dr\cong 0\ ;\ \ \varepsilon\sim\frac{1}{l}\cong\infty \tag{25}$$

To test the influence of this boundary condition data from previous channel flow calculation is used as a second type of inlet condition.

TESTCASE

A simple two dimensional test case from Nielsen (1990) is used to evaluate the calculation results. The inlet Reynolds-Number is Re = 5000, the turbulence intensity is 4%.

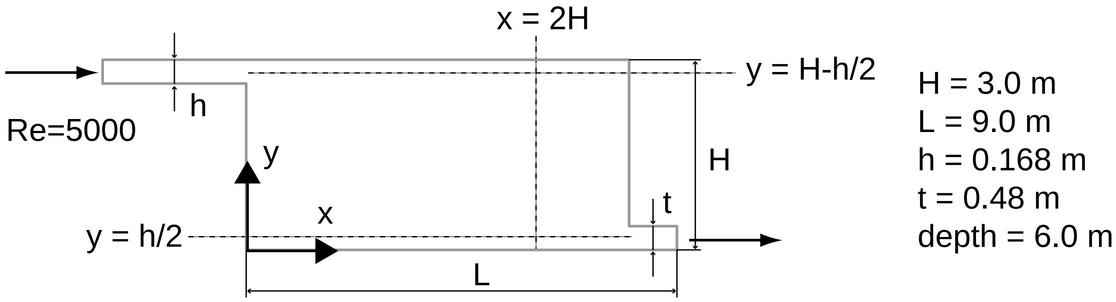

Figure 1: Two dimensional test case

RESULTS

Figure 2 shows the velocity profiles at x = 2H. All models predict the average velocity profile well. The KIM model gives a steeper gradient towards the top wall in agreement with the measurements. This result could be further improved by using channel flow data as inlet condition (KIM-CH). The turbulent fluctuation profiles indicate too low values for the KIM model with standard inlet boundary condition. The DAV models shows higher turbulent fluctuation velocity but only the KIM model with channel flow inlet data predicts the measured values.

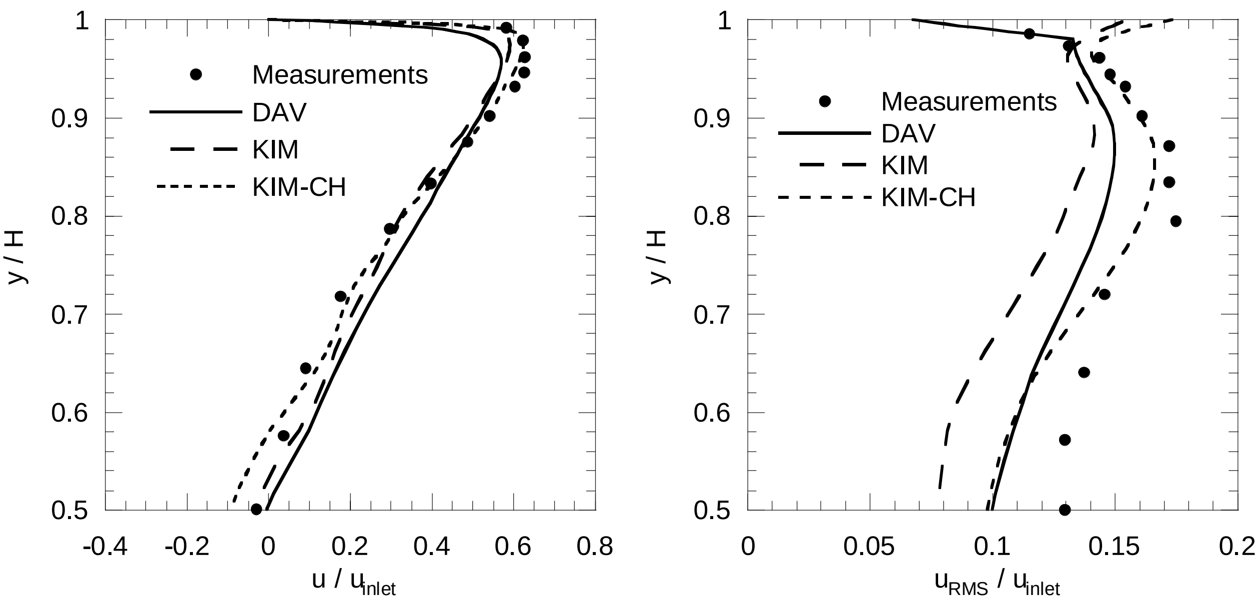

Figure 2: Velocity and turbulent fluctuation profiles at x = 2H, centre line

The profiles at y = H-h/2 are presented at Figure 3. Again, the KIM model with channel flow data shows the best agreement with the measured velocity profile. At y/H = 0 the inlet velocity is over predicted by the channel flow data because the room inlet device still shows entrance effects. The turbulent fluctuation profiles show the fast decay of standard inlet turbulence without any length scale. The calculation applying the channel flow data file preserves its turbulence level. The shape of the measured fluctuation profile is well predicted by the DAV model up to x/H = 2.5 and from the KIM model over the whole range. The level of the fluctuation velocities is to low between x/H = 2.5...3.

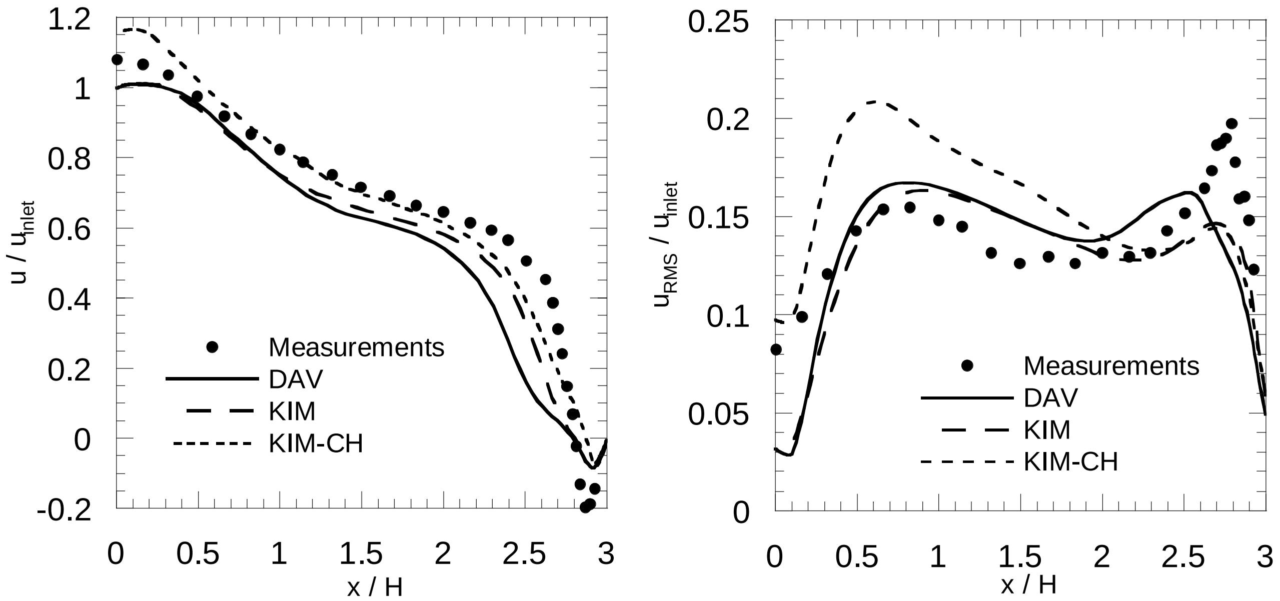

Figure 3: Velocity and turbulent fluctuation profiles at y = H-h/2, centre line

The velocity profile at y = h/2 is only well predicted by the KIM model with channel flow inlet data boundary condition. All other calculations give a to small recirculation zone of the wall jet. The calculated fluctuation velocities show a poor agreement with the measurements. The grid resolution in the lower half of the room is too low to preserve the resolved turbulent motion using a second order discretization method in space and time.

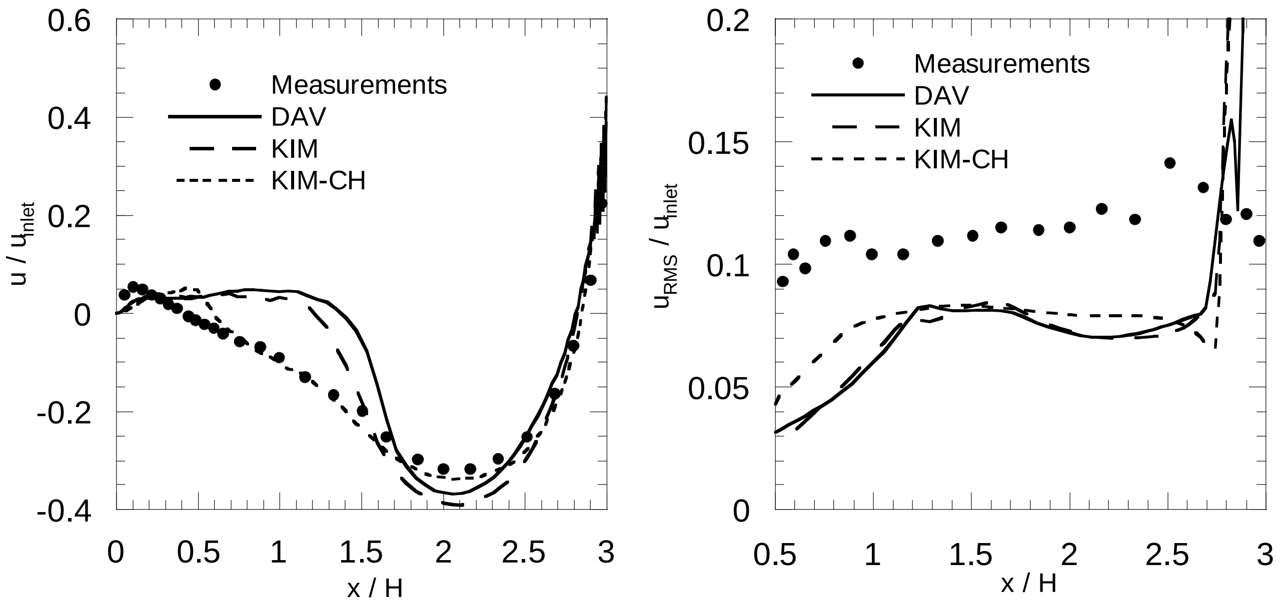

Figure 4: Velocity and turbulent fluctuation profiles at y = h/2, centre line

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