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REYNOLDS STRESS TRANSPORT MODELLING OF TRANSONIC FLOW AROUND THE RAE2822 AIRFOIL

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This paper presents a second moment closure Reynolds stress turbulence model implementation applied to a simulation of a well known transonic airfoil flow case (RAE2822). Comparisons are made between simpler turbulence models and the present implementation regarding its ability to predict the shock location and the shock-boundary layer interaction. To avoid explicitly added numerical dissipation in the mean flow equations, we take an approach where the Euler term, containing first derivatives, is discretized using the third order QUICK scheme. Comparisons are presented between a standard central scheme and the present approach. The results show that the Reynolds stress turbulence model is superior to the other turbulence models tested in predicting the shock location and the flowfield downstream of the shock, and that the implementation of the QUICK scheme significantly increases the accuracy on coarse meshes, while the need for explicitly added numerical dissipation is greatly reduced. For the simulations carried out in this work, no numerical dissipation was needed at all.

1 Introduction

Simpler turbulence models, like the Baldwin-Lomax[1] and the $k-\varepsilon$ models, which are widely used in Navier-Stokes solvers, often fail to accurately predict flow cases where large separation regions occur. Important cases are stall at high angles of incidence,[2] and separation and reattachment due to shock-boundary layer interaction. Davidson and Rizzi[3] have succeeded in predicting stall on an airfoil using an *algebraic* Reynolds stress model, and this paper investigates whether a more complex turbulence model is able to accurately predict the shock-boundary layer interaction. The second-moment closure model was implemented by Davidson[4] into a cell-centered finite-volume time marching Runge-Kutta code, originally written by Rizzi and Müller.[5] The Reynolds Stress transport equations are decoupled from the averaged flow equations and solved separately using a semi-implicit solver for the *steady* equations.[2] Comparisons are made with results from experiments and earlier implementations of simpler turbulence models into the same code.[6,2]

It is sometimes argued that the accuracy offered by advanced turbulence models is overwhelmed by the numerical dissipation, that has to be added to the averaged discretized Navier-Stokes equations when central differencing is used. Indeed, in our earlier work[3] we devised a function to turn off the artificial dissipation in the boundary layer in order to perserve the nature of the physical viscosity. This however is *ad hoc* and requires knowledge of where the wall is located. In this paper we take another approach, an upwind scheme with higher spatial resolution. We focus on the boundary layer where compressibility is negligible and discretize the inviscid flux terms with the QUICK scheme that is rather popular for incompressible flow. The intention was that the third-order truncation error term arising from the QUICK discretizations[7] would be small enough to not destroy the physical viscosity, but would be sufficient to stabilize the solution without the need for numerical dissipation. This scheme was then used in some numerical simulations, and our hypothesis was then tested by comparing the results with those obtained using the earlier implementation.[5]

2 Governing equations

2.1 The compressible Navier-Stokes equations

2.1.1 Averaging procedures

Turbulence modelling is based on the assumption that turbulent motion in a fluid can be described statistically. The fluid variables can, if this assumption is valid, be described as $\tilde{\Phi} = \overline{\tilde{\Phi}} + \phi$, where $\tilde{\Phi}$ is the instantaneous value of the variable, $\overline{\tilde{\Phi}}$ is the *time average* of $\tilde{\Phi}$ and ϕ is the fluctuating component, the time average of which, $\overline{\phi}$, is zero.

To make it clear what is meant by *time averaged*, the following definition is made: Let $\tilde{\Phi}$ be an arbitrary quantity varying in time. Then the time averaged quantity at time t_0 is defined by

$$\overline{\tilde{\Phi}}(t_0) = \frac{1}{2t}\int_{t_0-t}^{t_0+t} \tilde{\Phi}(\tau)d\tau \quad (1)$$

where t is a sufficiently long time period to include the largest time scales in the varying quantity.

There is, however an averaging procedure called *mass weighted averaging*, which is more convenient for compressible flows. Mass weighted averaging is defined by the following equations:

$$\tilde{\Phi} = \widehat{\Phi} + \acute{\phi} \quad (2)$$

$$\widehat{\Phi} = \frac{\overline{\tilde{\rho}\tilde{\Phi}}}{\overline{\tilde{\rho}}}; \quad \overline{\acute{\phi}} \neq 0 \quad (3)$$

The two averaging procedures are linked with the relation

$$\widehat{\Phi} = \overline{\tilde{\Phi}} + \frac{\overline{\acute{\rho}\acute{\phi}}}{\overline{\tilde{\rho}}} \quad (4)$$

For simplicity, throughout the paper the notation Φ is used instead of $\tilde{\Phi}$ where possible.

For the flow cases studied in this paper, it is assumed that the second term in equation (4) is small compared with the first term, and thus $\Phi \approx \widehat{\Phi}$.

2.1.2 Scaling of the variables

To make the flow variables functions of the freestream Mach-, Reynolds-, and Prandtl numbers only, they are made dimensionless by scaling as described in, for example [5].

2.1.3 The averaged Navier-Stokes equations

The Navier-Stokes equations for the averaged variables read

$$\frac{\partial}{\partial t}\int_V \Phi dV + \int_{\partial V} \left(F_i(\Phi) + F_i^v(\Phi)\right) n_i dS = 0 \quad (5)$$

where V is an *arbitrary* control volume with boundary ∂V and outward-pointing boundary unit normal n, and

$$\Phi = \overline{\tilde{\rho}}\begin{bmatrix} 1 \\ U_1 \\ U_2 \\ E \end{bmatrix}; F_i = U_i\Phi + P\begin{bmatrix} 0 \\ \delta_{i1} \\ \delta_{i2} \\ U_i \end{bmatrix} \quad (6)$$

$$F_i^v = -\begin{bmatrix} 0 \\ \Sigma_{i1}^{eff} \\ \Sigma_{i2}^{eff} \\ \overline{\tilde{\Sigma}_{ij}\tilde{U}_j} - \overline{\tilde{\rho}}\overline{u_i e} \end{bmatrix} - \kappa\begin{bmatrix} 0 \\ 0 \\ 0 \\ \Theta_{,i} \end{bmatrix} \quad (7)$$

and

$$E = \frac{1}{2}|U|^2 + c_v\Theta + k \quad (8)$$

$$P = (\gamma - 1)\overline{\tilde{\rho}}c_v\Theta \quad (9)$$

$$\Sigma_{ij}^{eff} = \mu\left((U_{j,i} + U_{i,j}) - \frac{2}{3}\delta_{ij}U_{m,m}\right) - \overline{\tilde{\rho}}\overline{u_i u_j} \quad (10)$$

$$k = \frac{1}{2}\overline{u_i u_i} \quad (11)$$

The correlation $\overline{\tilde{\Sigma}_{ij}\tilde{U}_j}$, the turbulent diffusion of energy, $\overline{\tilde{\rho}}\overline{u_i e}$, and the last term in equation (10), $\overline{\tilde{\rho}}\overline{u_i u_j}$, or the *Reynolds stress tensor*, are left for the turbulence models to provide.

2.1.4 Boundary conditions

The farfield boundary conditions are based on the theory of characteristics for the locally one-dimensional problem normal to the boundary. Details can be found in, for example, [6]. On the wall, the no-slip boundary condition is imposed for the velocity field, $U_i = 0$ and a homogenous Neumann boundary condition is imposed for the temperature, T, and the pressure, P.

2.1.5 Modelling of the energy equation

The modelling of the unknown correlations in the energy equation is in this work the same for both turbulence models used, and it is expressed as

$$\overline{\tilde{\Sigma}_{ij}\tilde{U}_j} - \overline{\tilde{\rho}}\overline{u_i e} + \kappa\Theta_{,i} = \Sigma_{ij}^{eff}U_j + \kappa^{eff}\Theta_{,i} \quad (12)$$

where a new quantity, the effective heat conductivity or κ^{eff}, has been introduced. The definition of this quantity is deferred to section 2.2.

The main approximation made in this model of the averaged energy equation is, that the turbulent diffusion of the turbulent kinetic energy, k, is considered small compared to the other terms in the equation and it is therefore neglected.

2.2 The eddy-viscosity assumption

Most of the simpler turbulence models do not explicitly calculate the Reynolds stress tensor, but use instead the mathematical concept of *eddy viscosity* or *turbulent viscosity*, μ_t. In the eddy-viscosity assumption, the definition of the stress tensor Σ_{ij}^{eff} in equation (10) becomes

$$\Sigma_{ij}^{eff} = (\mu + \mu_t)\left((U_{j,i} + U_{i,j}) - \frac{2}{3}\delta_{ij}U_{m,m}\right) \quad (13)$$

The effective heat capacity, κ^{eff}, left undefined above, is, when used together with the eddy-viscosity assumption defined as:

$$\kappa^{eff} = \frac{\gamma}{\gamma - 1}\left(\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t}\right) \tag{14}$$

where Pr_t, the *turbulent Prandtl number* is usually set to 0.9.

The only variable now left for the turbulence model to predict is the turbulent viscosity, μ_t.

2.3 The $k - \varepsilon$ model

In the $k - \varepsilon$ model, two transport equations are solved in addition to the averaged flow equations. One for the turbulent energy, k, and one for its dissipation rate, ε. The Reynolds stress tensor is then calculated using the *Boussinesq* assumption:

$$-\overline{\rho}\overline{u_i u_j} = \mu_t\left(U_{j,i} + U_{i,j} - \frac{2}{3}\delta_{ij}U_{m,m}\right) - \frac{2}{3}\delta_{ij}\overline{\rho}k \tag{15}$$

where the turbulent viscosity is given by

$$\mu_t = C_\mu \overline{\rho}\frac{k^2}{\varepsilon} \tag{16}$$

The constant C_μ is usually taken to 0.09.

Using the assumption that, for a turbulent quantity, $\hat{\tilde{\Phi}} \approx \Phi$, the transport equations for k and ε read:

$$\begin{aligned}\frac{\partial}{\partial t}\left(\overline{\rho}k\right) &= -\left(\overline{\rho}U_j k\right)_{,j} + \left(\left(\mu + \frac{\mu_t}{\sigma_k}\right)k_{,j}\right)_{,j} + \\ &\quad + P_k - \overline{\rho}\varepsilon \qquad (17)\\ \frac{\partial}{\partial t}\left(\overline{\rho}\varepsilon\right) &= -\left(\overline{\rho}U_j \varepsilon\right)_{,j} + \left(\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\varepsilon_{,j}\right)_{,j} + \\ &\quad + \frac{\varepsilon}{k}\left(c_{1\varepsilon}P_k - c_{2\varepsilon}\overline{\rho}\varepsilon\right) \qquad (18)\end{aligned}$$

where

$$P_k = -\overline{\rho}\overline{u_i u_j}U_{i,j} \tag{19}$$

and the turbulent Prandtl numbers are set to $\sigma_k = 1.0$ and $\sigma_\varepsilon = 1.3$. The constants $c_{1\varepsilon}$ and $c_{2\varepsilon}$ are set to $c_{1\varepsilon} = 1.44; c_{2\varepsilon} = 1.92$.

2.3.1 Near-wall treatment

The transport equations for k and ε are designed for high Reynolds-number flow. Therefore a special treatment is needed near the wall. Several near-wall models have been designed, and the one used here is a model by Wolfshtein,[8] modified by Chen and Patel.[9] It is a one-equation model, in which only the k-equation is solved. The dissipation rate, ε, and the turbulent viscosity, μ_t are then calculated using

$$\varepsilon = \frac{k^{\frac{3}{2}}}{l_\varepsilon}; \mu_t = C_\mu \overline{\rho}\sqrt{k}l_\mu \tag{20}$$

with the turbulent length scales, l_ε and l_μ, prescribed as

$$l_\varepsilon = c_l n\left(1 - e^{-\frac{-R_n}{A_\varepsilon}}\right); l_\mu = C_\mu n\left(1 - e^{-\frac{-R_n}{A_\mu}}\right) \tag{21}$$

with

$$R_n = \frac{\overline{\rho}\sqrt{k}n}{\mu} \tag{22}$$

and the constants defined as

$$C_\mu = 0.09 \quad c_l = \kappa C_\mu^{-\frac{3}{4}} \quad A_\mu = 70 \quad A_\varepsilon = 2c_l$$

The near-wall model is used out to a distance of order $n^+ = 50$ from the wall, and the matching line is, in this work, chosen along a preselected grid line. It should be noted, that the value of ε calculated using the near-wall model serves as a boundary condition for the transport equation for ε used in the outer field. Thus, a discontinuity in the value of ε cannot occur.

2.3.2 Boundary conditions

The far-field boundary conditions for k and ε are a homogenous Dirichlet boundary condition at the inflow boundary and a homogenous Neumann condition at the outflow boundary

The wall boundary condition for k is $k = 0$, and the near-wall boundary condition for the ε transport equation has been discussed above.

2.4 The Reynolds stress model

The compressibility effects are neglected and the modelled Reynolds stress equations become:

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\overline{\rho}\overline{u_i u_j}\right) = \\ &= -\left(\overline{\rho}U_k\overline{u_i u_j}\right)_{,k} - \qquad (C_{ij})\\ &-\overline{\rho}\left(\overline{u_i u_l}U_{j,l} + \overline{u_j u_l}U_{i,l}\right) + \qquad (P_{ij})\\ &+\Pi_{ij} - \overline{\rho}\epsilon_{ij} - D_{ij}\end{aligned} \tag{23}$$

2.4.1 Second moment closure

The dissipation term, (ϵ_{ij}), the diffusion term, (D_{ij}), and the pressure-strain term (Π_{ij}), which promotes isotropy turbulence, all include unknown correlations and are modelled in a standard way.[10]

Two different ways of modelling the diffusion term, (D_{ij}), have been tested. These are:

$$(D_{ij}) = -\left(-c_s\frac{k}{\varepsilon}\overline{\rho}\overline{u_k u_l}\left(\overline{u_i u_j}\right)_{,k} + \mu\left(\overline{u_i u_j}\right)_{,l}\right)_{,l} \tag{24}$$

$$(D_{ij}) = -\left(\left(\mu + \frac{\mu_t}{\sigma_\mu}\right)\left(\overline{u_i u_j}\right)_{,l}\right)_{,l} \tag{25}$$

the latter expression being somewhat simpler than the first. Both models have been used in this work; the closure model using equation (24) is hereafter called RSM-GGDH, where

GGDH stands for "Generalized Gradient Diffusion Hypothesis", and the closure model using equation (25) is simply called RSM.

The constants in the model are

$$c_1 = 1.8; \quad c_2 = 0.6; \quad c_1' = 0.5; \quad c_2' = 0.18;$$

$$C_\mu = 0.09; \quad \sigma_\mu = 1; \quad c_s = 0.22$$

2.4.2 Modification of the $k-\varepsilon$ equations

In addision to the Reynolds Stress transport equations, the k- and ε transport equations have also to be solved. They have the same form as discussed in Section 2.3, with the exception that the production term, P_k now uses the Reynolds stress tensor calculated from the RSM equations instead of the Boussinesq assumption in the *outer* region. In the inner region, where the one-equation model is used, the Boussinesq assumption is still used.

In the RSM-GGDH closure model, the diffusion in the k and ε equations is also modelled using the *generalized gradient diffusion hypothesis.* The k and ε equations then take the form

$$\begin{aligned} \frac{\partial}{\partial t}\left(\bar{\rho}k\right) &= -\left(\bar{\rho}U_j k\right)_{,j} + \\ &+\left(\left(\mu\delta_{jm} + c_k\frac{k}{\varepsilon}\overline{\bar{\rho}u_j u_m}\right)k_{,m}\right)_{,j} + P_k - \bar{\rho}\varepsilon \end{aligned} \tag{26}$$

$$\begin{aligned} \frac{\partial}{\partial t}\left(\bar{\rho}\varepsilon\right) &= -\left(\bar{\rho}U_j \varepsilon\right)_{,j} + \\ &+\left(\left(\mu\delta_{jm} + c_\varepsilon\frac{k}{\varepsilon}\overline{\bar{\rho}u_j u_m}\right)\varepsilon_{,m}\right)_{,j} + \\ &+\frac{\varepsilon}{k}\left(c_{1\varepsilon}P_k - c_{2\varepsilon}\bar{\rho}\varepsilon\right) \end{aligned} \tag{27}$$

$$c_k = 0.22; c_\varepsilon = 0.17$$

2.4.3 Near-wall treatment

The modelling of the unknown correlations in the previous section is designated for high Reynolds number flow. Therefore, in the vicinity of a wall, the one-equation model discussed in Section 2.3.1, is used, and the Reynolds stress tensor is then calculated using the Boussinesq assumption.

The matching line, inside of which the one-equation model is used instead of the RSM equations, is in this work chosen to be the same as in the $k-\varepsilon$ model. In this way, the Reynolds stress tensor, calculated by the Boussinesq assumption and the one-equation model, serves as a boundary condition for the RSM transport equations at the matching line.

2.4.4 Farfield boundary conditions

For the components of the Reynolds stress tensor, a homogenous Dirichlet condition is applied on the inflow boundary and a homogenous Neumann condition on the outflow boundary.

3 The solver

3.1 Solution procedure

When the $k-\varepsilon$ solver was first implemented in this code,[2] attempts were carried out to couple the $k-\varepsilon$ transport equations to the averaged flow equations and solve the whole system with a finite-volume time-marching Runge-Kutta procedure. However, no stable convergent solution could be obtained, although a recent paper shows that this is possible.[11] Instead a semi-implicit solver, well known from the *SIMPLE*[12] algorithm was implemented to solve these transport equations. This method is also used to solve the transport equations for the Reynolds stress tensor. The solution strategy becomes:

* Obtain Start-approximation for *all* variables.

REPEAT

* Perform a Runge-Kutta timestep for the averaged flow variables using old values of the Reynolds stress tensor and effective heat conductivity.
* Perform an iteration of the *steady* ε transport equation solver using old values of k, and the Reynolds stress tensor and new values of the averaged flow variables.
* Perform an iteration of the *steady* k transport equation solver using old values of the Reynolds stress tensor and new values of the averaged flow variables and ε.
* Perform an iteration of the *steady* $\overline{u^2}$ transport equation solver using old values of the correlations $\overline{v^2}$ and $\overline{uv}$ and new values of k, ε and the averaged flow variables.
* Perform an iteration of the *steady* $\overline{v^2}$ transport equation solver using old values of the correlation $\overline{uv}$ and new values of k, ε, $\overline{u^2}$ and the averaged flow variables.
* Perform an iteration of the *steady* $\overline{uv}$ transport equation solver using new values for all quantities.

UNTIL CONVERGENCE

In the above solution method description, *old* values are values that are obtained from the previous iteration and *new* values are values that are obtained earlier in the iteration.

3.2 Space discretization

3.2.1 The finite-volume approximation

As seen in equation (5) the Navier-Stokes equations for the averaged flow variables are on integral form, valid for an