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Role of Initial Conditions in Establishing Asymptotic Flow Behavior

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The role of the details of initial (or upstream) conditions in establishing asymptotic behavior is reviewed. The traditional view that at least simple shear flows reach universal asymptotic states is shown to be inconsistent with the abundant experimental and DNS data. Even though scaled mean velocity profiles collapse, the streamwise (or temporal) variation of the scaling parameters and spreading rates can vary widely for different upstream (or initial) conditions. Equilibrium similarity theory shows why the traditional view has arisen: normalized mean velocity profiles can be universal, even though the other moment profiles and scaling parameters are not. Decaying isotropic turbulence and applications of the POD to free shear flows are used to show what parameters might control the downstream development. It is argued that RANS cannot account for these dependencies, but LES can.

Introduction

One of the most persistent ideas from the past century of turbulence research is that turbulence 'forgets' its initial conditions. Indeed, most mean velocity profiles of simple free shear flows collapse when plotted using a velocity scale and an appropriately defined width. For example, the normalized mean velocity profiles shown in Figures 1 – 3 for the far axisymmetric wake (in a uniform external stream) from a variety of wake generators (including DNS) all collapse to virtually the same curve, seemingly independent of initial (or upstream) conditions (Johansson et al. 2003). (U_o is the centerline velocity deficit given by $U_\infty - U_{CL}$ and δ_* is defined to make the integral of equation 11 unity.)

Plots like this are shown in most texts, and cited as evidence for asymptotic independence of initial conditions. What is often not shown — even in many of the original journal papers — is the differing streamwise dependence of the normalizing (or scaling) parameters from experiment to experiment, or from one source to another. Figure 4 shows the wake half-widths (the normalization length scale for the plots above) for the same experiments shown in Figures 1 – 3, along with a number of other experiments. Clearly the spreading rates of these wakes depend dramatically on the initial conditions to distances very far downstream, and by amounts that can not be simply attributed to experimental error. This is contrary to the conventional wisdom; so either the traditional view must wrong, or it hardly provides useful information since the transients persist so far downstream.

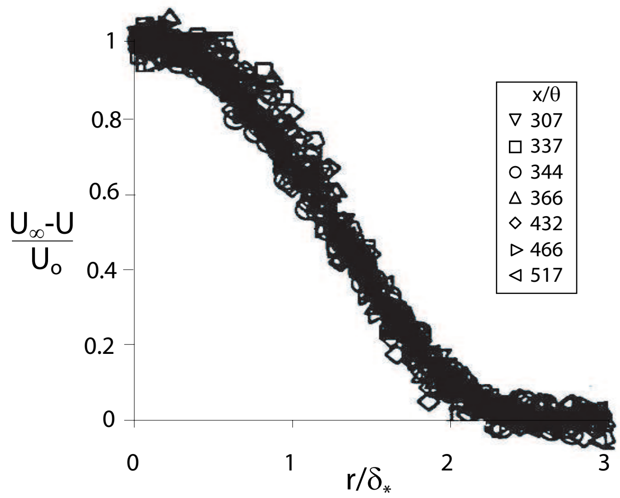

Fig. 1 Mean velocity profiles for the porous disk with σ = 0.70, data from Cannon (1991). (Adapted from Johansson et al. 2003).

This paper will briefly attempt to assess what we know at present about the role initial conditions play in turbulence. Our focus is not on LES *per se*, but rather to review our current understanding of the underlying physics. Our hope is that this will contribute to a better understanding of how to use LES for real problems, and as well stimulate its use to explore further these phenomena.

How can some profiles be independent of initial conditions, and others not?

There are numerous experiments and computer simulations over the past decade and a half which confirm the apparent asymptotic dependence of spreading rate

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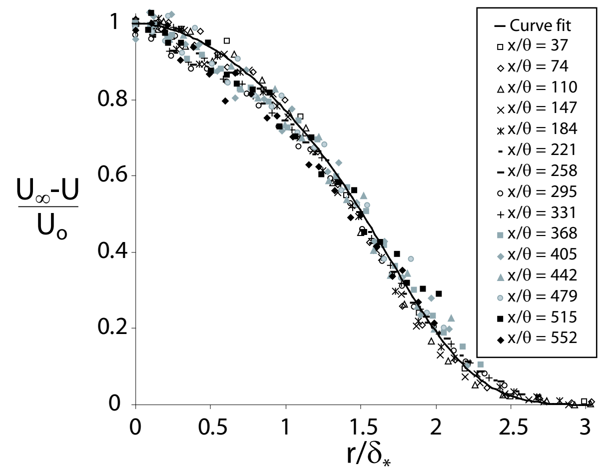

Fig. 2 Mean velocity profiles for disk from Johansson (2002), (Adapted from Johansson et al. 2003).

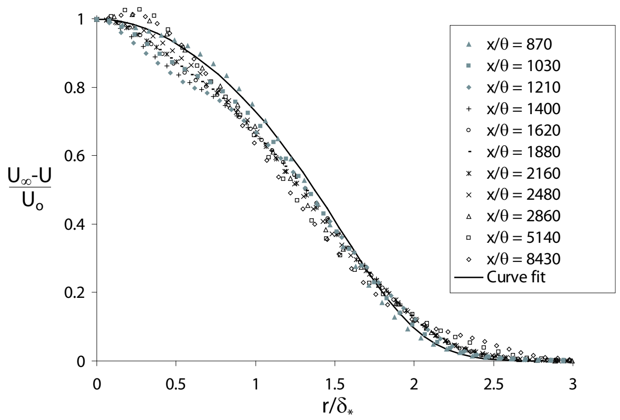

Fig. 3 Mean velocity profiles for the DNS high Reynolds number wake data of Gourlay (2001). Solid line shows the fit in Figure 2 to the data of Johansson (2002) above. (Adapted from Johansson et al. 2003).

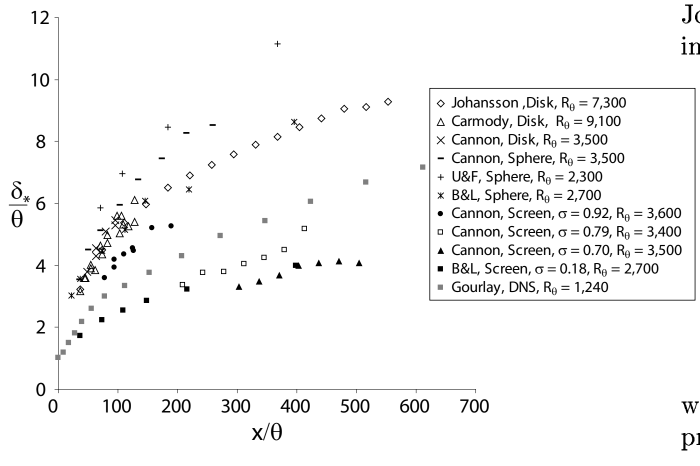

Fig. 4 Cross-stream length scale, δ_*/θ versus x/θ. For the screen wakes, the porosity, σ, is defined as σ = (solid area)/(total area). (Adapted from Johansson et al. 2003).

of free shear flows on the initial conditions. These include Wygnanski et al. (1986) (experiments in plane wakes), Grinstein et al. (1995) (DNS and LES of jets), Ghosal and Rogers (1997) (LES of wakes), Boersma et al. (1998) (DNS of jets), Mi et al. (2001) (experiments in jets), Slessor et al. (1998) (experiments in shear layers), to cite but a few. The wake experiments used different wake generators, the jets different exit conditions, and the LES and DNS different forcing of the largest scales. While the variations in jet spreading rate were on the order of 20% typically, it was possible to make dramatic differences in the wakes (several hundred percent or more). There is even some evidence that the outer part of turbulent boundary layers retains a dependence on its upstream conditions (George et al. 2000, Castillo and Johansson 2000, Castillo and George 2001).

Given the long history of turbulence, it is not enough to simply state that a previously believed idea is wrong, or even that another idea is better. Instead it is necessary to show why it came to be believed in the first place. The point of this section is to outline briefly how this happened, and in the process show what was overlooked.

As will be discussed below, it is not clear at this time precisely *how* or *why* the initial conditions can be preserved asymptotically. It has been clear since George (1989), (1995), however, that *equilibrium similarity analysis* can account for this behavior. In other words, there is no conflict between properly done similarity theory and these results. The initial conditions simply show up in the spreading rate and the undetermined coefficients. Most importantly, if the Reynolds stress and mean velocity are properly normalized, the mean momentum equation is independent of all parameters. For example, for the high Reynolds number axisymmetric wake in a constant velocity external flow (cf. Johansson and George 2003), the momentum equation in similarity variables reduces to:

$$\left[\frac{\delta}{U_s}\frac{dU_s}{dx}\right] f - \left[\frac{d\delta}{dx}\right]\eta f' = \left[\frac{R_s}{U_\infty U_s}\right]\frac{1}{\eta}\frac{\partial}{\partial\eta}\eta g \qquad (1)$$

where any single point moment is assumed to be expressible as a product of two functions[1]; i.e.,

[1]This is not restrictive, since if it is not possible, no solutions will be found

$$U - U_\infty = U_s(x,*)f(\eta,*) \quad (2)$$
$$-\langle uv\rangle = R_s(x,*)g(\eta,*) \quad (3)$$
$$\frac{1}{2}\langle u^2\rangle = K_u(x,*)k_u(\eta,*) \quad (4)$$
$$\frac{1}{2}\langle u^2 v\rangle = T_{u^2v}(x,*)t_{u^2v}(\eta,*) \quad (5)$$
$$\langle \frac{p}{\rho}\frac{\partial u}{\partial x}\rangle = P_u(x,*)p_u(\eta,*) \quad (6)$$
$$\epsilon_u = D_u(x,*)d_u(\eta,*) \quad (7)$$
$$etc. \quad (8)$$

and where $\eta = r/\delta(x,*)$ and the '*' denotes a possible (but unknown) dependence on initial (or upstream) conditions. U_s can be taken to be the centerline velocity deficit, U_o, and δ can be taken as the δ_* defined below with no loss of generality.

Thus we are seeking solutions to transformed versions of the momentum (and Reynolds stress equations as well) for which the bracketed terms are functions of only the streamwise variable, x, and the upstream conditions, and the functions outside the square brackets are functions of the similarity variable, η, only. It is very important to also note that we have allowed each moment entering the equations to have its own scaling function and we have not specified them arbitrarily nor by *ad hoc* arguments. For example, we have NOT taken $R_s(x) = U_s(x)^2$, etc. as in most texts.

Now the question we ask is: do there exist solutions to these transformed equations (or subsets of them) for which all of terms in square brackets have exactly the same dependence on x? If so, we call these *equilibrium similarity solutions.* It is clear they must be the asymptotic solution, since if any term has a different x-dependence, it will either dominate or die off leaving a different set of equations. For our example, equilibrium similarity requires:

$$\left[\frac{\delta}{U_s}\frac{dU_s}{dx}\right] \quad \propto \quad \left[\frac{d\delta}{dx}\right] \quad \propto \quad \left[\frac{R_s}{U_\infty U_s}\right] \quad (9)$$

This implies that an equilibrium similarity solution is possible only if the scale for $-\langle uv\rangle$ is:

$$R_s \propto U_\infty U_s \frac{d\delta}{dx} \quad (10)$$

It is also straightforward to show that equation 10 implies U_s must be proportional to a power of δ — but not necessarily a power of x, contrary to the assumption of most texts. The explicit x-dependence can only be found by consideration of the higher moment equations (cf. George 1989, 1995 or for this problem Johansson et al. 2003).

Almost every problem has some kind of integral constraint; for this one the net momentum deficit must equal the drag. Far downstream this implies that:

$$U_\infty U_s \delta^2 \int_0^\infty f\eta d\eta = U_\infty^2 \theta^2 \quad (11)$$

where θ is the momentum thickness which is constant for the wake in a uniform stream. Note that δ_* is defined as the choice of δ which makes the integral unity. Since the integral can at most depend on the initial (or upstream) conditions, it follows from differentiation that:

$$\frac{\delta}{U_s}\frac{dU_s}{dx} = -2\frac{d\delta}{dx} \quad (12)$$

Substitution into equation 1 yields:

$$-2f - \eta f' = \left[\frac{R_s}{U_\infty U_s d\delta/dx}\right]\frac{1}{\eta}\frac{d}{d\eta}\eta g \quad (13)$$

But this can be reduced even further by incorporating the scale factor $R_s/(U_\infty U_s d\delta/dx) = constant$ into g by defining:

$$\tilde{g} = \left[\frac{R_s}{U_\infty U_s d\delta/dx}\right] g \quad (14)$$

from which it follows that the momentum equation reduces to:

$$-\frac{d}{d\eta}\eta^2 f = \frac{d}{d\eta}\eta\tilde{g} \quad (15)$$

Thus the final scaled momentum equation is entirely independent of the initial conditions, no matter how much the scaling parameters themselves depend on them.

Since the scaled mean velocity profile is the solution to equation 15, obviously it must be independent of initial conditions as well. By contrast, the scaling parameters, U_o and $R_s = U_\infty U_s d\delta/dx$, can still depend on source conditions (and from all evidence do). The Reynolds stress is especially interesting since it is normalized by some combination of velocity squared times the growth rate $d\delta/dx$ (instead of just the U_s^2 found in most journal articles and texts). Thus the Reynolds stress *profile* is the same for all wakes, but its amplitude depends on the initial conditions. It is this incorporation of the growth rate into the Reynolds stress scaling that removes the initial conditions from the transformed momentum equation.

As first noted by George (1989), this absorbtion of the initial condition dependent parameters can usually only be done for the mean momentum equation and not for the higher moment equations. Thus the higher moment profiles can vary from one set of initial conditions to another. Interestingly, all the normal stresses, $\langle u^2\rangle$, $\langle v^2\rangle$ and $\langle w^2\rangle$, can be shown to scale with U_s^2, but all the transverse transport moments involve $d\delta/dx$ (like $\langle uv\rangle$ which scales with $U_\infty U_s d\delta/dx$ instead of simply U_s^2). The same is also true for the third moments where the lateral transport moments all involve $d\delta/dx$. Physically this makes sense, since it is the lateral transport of momentum and energy that allows the flow to spread. If the spreading rate depends on the initial conditions, then so must the scaling parameters for these moments — and they do.

Equilibrium similarity has been discussed in detail in a variety of places in addition to the two George references; e.g., Moser et al. (1998), Rogers (2002), Johansson et al. (2003), George et al. (2000) Castillo and George (1997,2001). The important point to be drawn from the discussion here is that in every case, the result is the same: the scaled momentum equation implies that the *properly scaled* mean velocity and Reynolds stress profiles are always independent of the initial conditions, but scaling parameters and other moment profiles are not. **Thus collapse of some profiles (like mean velocity) has nothing to do with whether the turbulence retains a dependence on initial conditions.** Instead it is an artifact of the choice of scales and the momentum equation itself. But it was precisely this universality of such mean profiles that led the turbulence community to accept the idea of independence from initial conditions in the first place — clearly a logical fallacy. And it was the erroneous assumption that all moments could be scaled by a single length and velocity scale (self-preservation) that seemed to justify it.

Homogeneous turbulence remembers too.

The asymptotic dependence on initial conditions is not confined to the single point RANS equations of turbulent free shear flows, but characterizes the multipoint equations as well (e.g., George, 1992, George and Gibson 1992, Ewing 1995, Ewing and George 1995). For example, equilibrium similarity theory can be applied to the spectral energy equation of *decaying* isotropic turbulence:

$$\frac{\partial E}{\partial t} = T - 2\nu k^2 E \tag{16}$$

where $E(k,t)$ is the three-dimensional spectrum function and $T(k,t)$ is the non-linear spectral transfer function. George 1992 was able to *deduce* that equilibrium similarity solutions were possible of the form:

$$E(k,t) = E_s(t,*)F(\overline{k},*) \tag{17}$$

and

$$T(k,t) = T_s(t,*)G(\overline{k},*) \tag{18}$$

where $\overline{k} = kl(t)$ and $l(t)$ could be either the integral scale or the Taylor microscale (since their ratio was constant during decay). Figures 6 and 5 from the paper by Wang and George (2002) show such collapse for the data from two recent 512^3 DNS for decaying

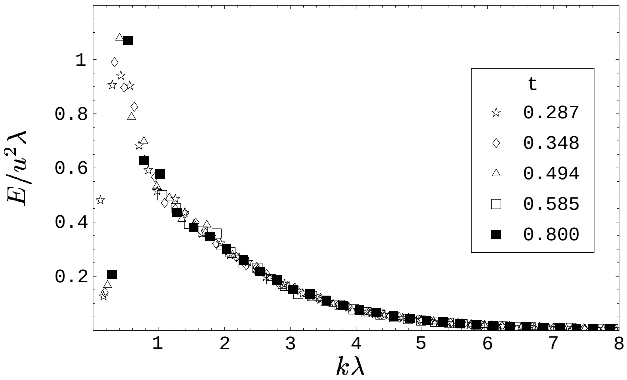

Fig. 5 **$E(k)/u^2\lambda$ versus $k\lambda$ for the de Bruyn Kops/Riley 1999 DNS data. Adapted from Wang and George (2002)**

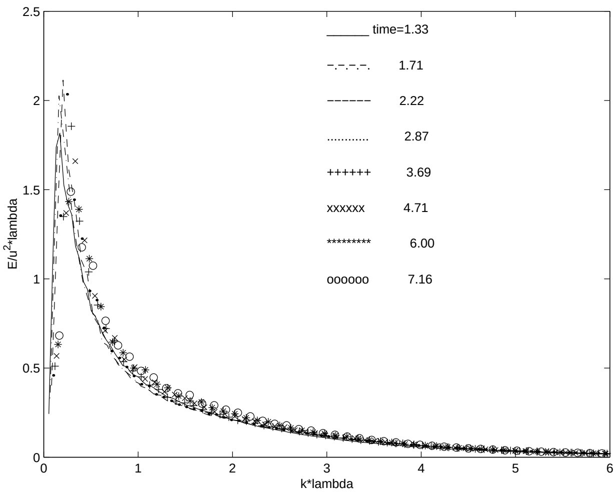

Fig. 6 **$E(k)/u^2\lambda$ versus $k\lambda$ for the Wray 1998 DNS data. Adapted from Wang and George (2002)**

turbulence using the deduced scales, $E_s = u^2\lambda$ and $T_s = \nu u^2/\lambda$.

The theory also deduces that the energy should decay as a power law in time, $u^2 \propto t^{-n}$, where the constant decay exponent n could possibly depend on the initial conditions. For all data sets considered (including the experimental data of Comte-Bellot and Corrsin 1966, 1971) it did.

The non-dimensional non-linear spectral transfer term, $G(\overline{k})$, is also invariant during decay and depends uniquely on the decay exponent, n, and the energy spectrum, $F(\overline{k})$. It is straightforward to show that equilibrium similarity applied to equation 16 yields:

$$G = \left\{ \left[\frac{5}{n}\right] (\overline{k}F' + F) - 10F \right\} \quad + \quad 2\overline{k}^2 F \tag{19}$$

Figure 7 shows the non-linear spectral transfer for a single time ($R_\lambda = 50$) from the DNS simulation of Wray (1998) and that calculated from equation 19 using the energy spectrum function and the value of

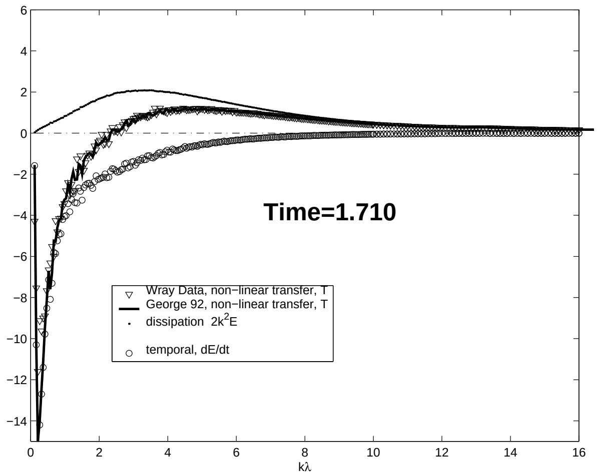

Fig. 7 DNS data of Wray (1998) for $\lambda T/\nu u^2$ *vs* $k\lambda$ with terms calculated from similarity equation 19 using measured spectrum: $n = 1.5$, $R_\lambda = 50$. Adapted from George and Wang (2002).

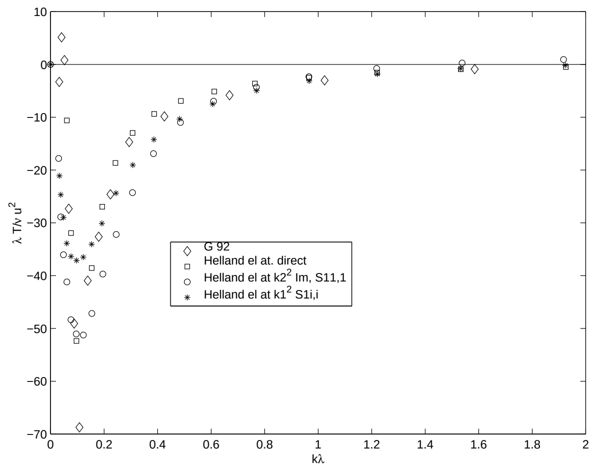

Fig. 8 Experimental data of Helland et al. (1977) for $\lambda T/\nu u^2$ *vs* $k\lambda$ with similarity equation 19 using measured spectrum: $n = 1$ and $R_\lambda = 237$. Adapted from George and Wang (2002).

$n = 1.5$ determined from the energy decay rate by Wang and George (2002a, 2000b). Also shown separately are the dissipation and temporal decay spectra, the latter of which is clearly not negligible as often assumed. Figure 8 shows the same calculation for the high Reynolds number experiment ($R_\lambda = 237$) of Helland et al. (1977) at a single downstream distance from a grid using $n = 1$. The value of n is crucial in both figures in determining the low wavenumber behavior and locating the minimum value.

Figure 9 is adapted from George (1992) and suggests strongly that the decay exponent for turbulence behind grids depends mostly on the Reynolds number of the grid. In fact, there is no evidence that the turbulence ever 'evolves' into the so-called 'final period of decay' for which $n = 5/2$, but rather it decays this

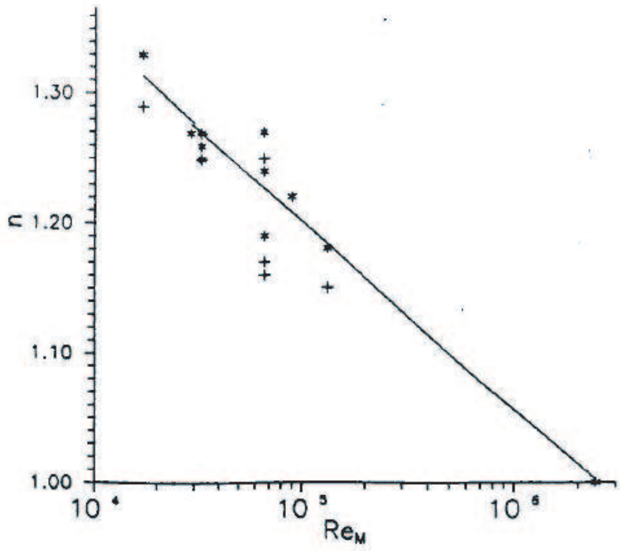

Fig. 9 Dependence of decay exponent on grid Reynolds number (from George 1992).

way when the Reynolds number of the grid is very low. (Note that the value of n is constant throughout decay, and departures from constant n are most likely due to box size effects as the scales grow too large for the experimental or computational domain (cf., Wang and George 2000.)

Much stronger variation of n with initial conditions has been observed for DNS (e. g. Wray and Mansour 1994, Wang and George 2002). In fact, for at least the modest to low Reynolds numbers of the DNS and experiments, the exponent n appears to be determined by the very lowest wavenumbers of the spectrum which are invariant (in physical variables) during decay. In particular, if $E(k,t) = C_p k^p$ for small k and they are invariant during decay, then:

$$n = \frac{p+1}{2} \tag{20}$$

The current thinking is that $0 < p \leq 4$, which would imply to $1/2 < n \leq 5/2$. (Note that there is no reason to assume p to be integer, nor the spectrum to be analytical for small k.) The recent shell-model simulations of Melander (2002) support the overall equilibrium similarity, but suggest strongly that equation 20 is only valid for relatively small values of R_λ. Clearly this means the lowest wavenumbers are not invariant (in physical variables) at high initial Reynolds numbers, but are at low initial Reynolds numbers. Although we do not yet completely understand this, it is consistent with the observation for grid turbulence that the value of n diminishes towards unity as the grid Reynolds number is increased.

Does it matter that turbulence can remember?

At first sight it might seem that it does not matter. After all we are simply solving differential equations and of course we should expect them to depend on the initial and boundary conditions. Actually the question is of considerable importance both to LES and the study of turbulence in general. If inlet boundary conditions uniquely determine the asymptotic solution, without this knowledge we have no idea how to choose them other than blindly. But there is an even more serious problem, first noted by Taulbee (1989) and George (1989), both in the same volume. Single point turbulence models (RANS) are missing the necessary physics to be able to account for these asymptotic effects of the initial conditions. This is most easily illustrated by two examples.

Example 1: Simple eddy viscosity

First consider a simple eddy viscosity which relates Reynolds stress to the mean velocity gradient for the wake considered above; say,

$$\langle -uv \rangle = \nu_t \frac{\partial U}{\partial r} \tag{21}$$

where we will take $\nu_t = C_\mu U_s \delta$.

In similarity variables this becomes:

$$[R_s]\, g = \left[\frac{\nu_t U_s}{\delta}\right] \frac{df}{d\eta} \tag{22}$$

As before all the x-dependence is in the square brackets. Using $R_s = U_\infty U_s d\delta/dx$, it is clear that ν_t must satisfy the following:

$$\nu_t = C_\mu U_s \delta \propto U_\infty \delta \frac{d\delta}{dx} \tag{23}$$

But we know $d\delta/dx$ depends on the initial conditions. Hence the initial conditions that affect the asymptotic growth rate show up in the worst possible place — the modelling coefficient.

It is easy to show that the same line of reasoning applies to any kind of gradient closure. For example, if we take $\langle -u^2 v \rangle \propto \partial \langle u^2 \rangle / \partial r$, the same problem occurs: we end up with our modelling coefficient proportional to $d\delta/dx$. Thus even if the model predicts exactly the right *dimensionless* profile (and it usually does), the spreading rate is entirely determined by the model coefficients. And these are in turn determined by some unspecified (and as yet unknown) source conditions.

Example 2: Isotropic decaying turbulence

It is straightforward to show that for the isotropic decaying turbulence considered above, equilibrium similarity implies directly (without additional assumptions) that:

$$\frac{dk}{dt} = -\epsilon \tag{24}$$

and surprisingly,

$$\frac{d\epsilon}{dt} = -\left[\frac{n+1}{n}\right]\frac{\epsilon^2}{k} = -C_{\epsilon_2}\frac{\epsilon^2}{k} \tag{25}$$

where k is the kinetic energy, ϵ is the dissipation, n is the constant power law exponent. Thus equilibrium similarity yields exactly the familiar k-ϵ model, but as an exact result and not a model! Moreover the coefficient, C_{ϵ_2}, in the ϵ-equation is uniquely determined by the the decay exponent, n, which appears to be dependent on the initial conditions. In this case, however, we have a clue that it depends on the initial spectrum at the lowest wavenumbers (at least at low Reynolds numbers). If so, then our RANS model may be exact, but it is virtually useless since there is nothing in the RANS equations to determine the unknown p (or n) without specifying it in advance.

But doesn't LES have the same problem?

Actually the answer appears to be no. LES, whether in space variables or wavenumber variables, resolves the largest scales (or lowest wavenumbers) exactly — at least within the limitations of the box-size (or lowest wavenumber). And it seems to be these which dictate the most important initial conditions. Two examples illustrate this.

The first is the example of isotropic decaying turbulence above. If the overall decay rate is truly determined by the lowest wavenumbers (at least for modest Reynolds numbers), then it is precisely these that the LES keeps. Clearly this puts substantial constraints on the ratio of the energetic scales to these very large scales, perhaps similar to those noted by Wang and George (2002) for the integral scale. Obviously unphysical backscatter from the closure model can also have a detrimental effect.

The second example is the LES simulation of the time-dependent wake by Ghosal and Rogers (1997) cited above. They first showed they could duplicate the DNS results reported by Moser et al. (1998) for the unforced time dependent wake. Then they showed by applying varying amounts of forcing to the largest scale they could vary the asymptotic growth rate substantially (by factor of 5), all the while maintaining equilibrium similarity.

How does turbulence remember?

From the two examples above it is clear, both in principle and in practice, that LES contains the 'necessary physics' to produce an asymptotic dependence on initial conditions. Similarly it is clear that RANS does not. Even though the RANS models may have exactly the right functional dependence, the fact that the initial conditions appear in the coefficients renders them fundamentally flawed. In particular, no general set of universally valid parameters should be expected. Note that this, of course, does not preclude their careful use for engineering purposes, especially where empirical knowledge is built into these coefficient choices. This is, of course, exactly what we have learned from two decades of RANS use.

But what is this 'necessary physics'. Obviously it must be related to vorticity production, convection and diffusion. But word descriptions of what is happening alone are not enough to explain how the effects can persist far downstream. Clearly there must be some means by which whatever structures are there can be continuously regenerated, and in a manner which retains some of the initial conditions. How this might happen poses very difficult questions to which we are just beginning to learn some clues. Some come from the recent POD studies of Delville et al. 1999), Citriniti and George (2000), Gordeyev and Thomas (2000), Ukeiley et al. (2001), Jung et al. (2001), Johansson et al. (2002), Gamard et al. (2002), George et al. (2002), among others. These allow a detailed examination of how the turbulence evolves in space and time. Moreover the POD results can be used to build dynamical models for the turbulence, which distinguishes them from many other approaches. In the following paragraphs we will summarize some recent experimental results, with especial attention to the axisymmetric wake. (For more detail, please consult the original references or George et al. (2002) on which this section is based.)

Johansson et al. (2002) report application of the 'slice' POD using rakes of hot-wires at various downstream cross-sections of the axisymmetric wake behind a disk at a Reynolds number based on diameter and free stream velocity of 28,000. Jung et al. (2001) and Gamard et al. (2002) report similar application of the slice POD using 138 hot-wires in the axisymmetric jet with nearly top-hat source conditions at source Reynolds numbers ranging from 40,000 to 157,000. In both experiments the first radial POD mode contains approximately 60 % of the resolved streamwise energy (about 40 % of the total streamwise energy), and only it will be of interest herein.

Of primary interest is the eigenspectrum of this first POD mode which shows how the energy of the cross-section is distributed with azimuthal mode number, m, and temporal frequency, f; i.e., $\lambda^{(1)}(m,f)$. Note that the variable f is considered to be continuous, while m is integer and positive only for the eigenspectra considered. The eigenspectra can be integrated over frequency, f, to obtain the distribution of energy with only the azimuthal mode number, m. If this is normalized by the total energy at the cross-section the result is:

$$\xi^{(1)}(m) = \frac{\int_0^\infty \lambda^{(1)}(m,f)\,df}{\sum_{m=0}^{M}\int_0^\infty \lambda^{(1)}(m,f)\,df} \qquad (26)$$

where M is the highest resolved azimuthal mode. Figure 10 shows the downstream evolution of this first azimuthal eigenspectrum, $\xi^{(1)}(m)$, for the jet.

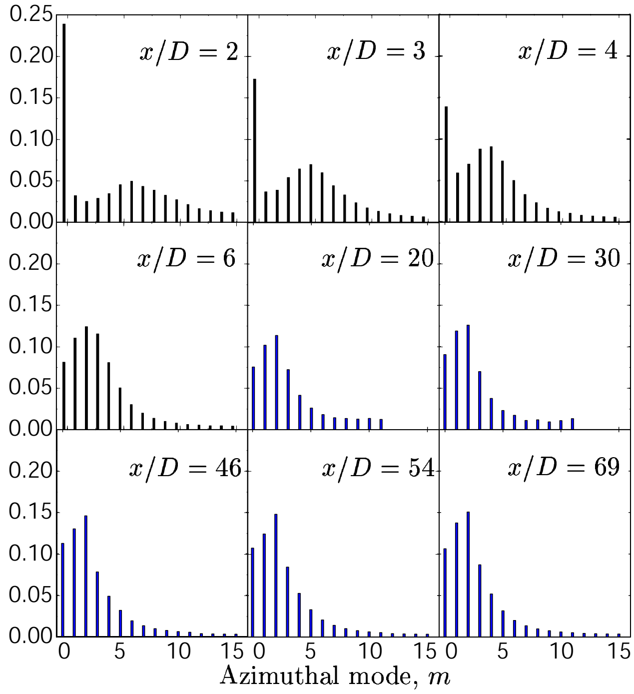

Fig. 10 Energy distribution with azimuthal mode number for the lowest order POD mode of the axisymmetric turbulent jet. From Gamard (2002).

For the jet, the azimuthal mode distribution in Fig. 10 at $x/D = 2$ shows a dominant peak at mode-0, and a distribution of energy centered about mode-0. As the distance from the exit plane is increased, mode-0 diminishes and the center of the distribution moves to lower values, from mode-5 at $x/D = 3$ to only a distribution around mode-2 by $x/D = 6$. After $x/D \approx 6$, the distribution shows no further evolution, coincident with the fact that the mean centerline velocity has approximately reached near similarity behavior (about $x/D \approx 10$). Also the ratio of centerline rms velocity to the mean centerline velocity is constant shortly after this evolution is complete, and the mean velocity profiles and turbulence intensity profiles begin to collapse as well. Obviously the diminishing value of mode-0 and the emergence of the mode-2 peak both reflect (or are responsible for) the process by which a top-hat profile evolves into a self-preserving jet.

Figure 11 shows a similar azimuthal mode evolution for the axisymmetric wake behind a disk. For the near wake, at $x/D = 10$, mode-1 dominates, but by $x/D = 30$, the energy in mode-2 is nearly equal to that in mode-1. By $x/D = 50$, mode-2 dominates, as it does for all downstream positions.

Like the jet, the emergence of this mode-2 dominance corresponds also to the emergence of the similarity state, particularly evident in the normalized turbulence intensity which does not approach a con-

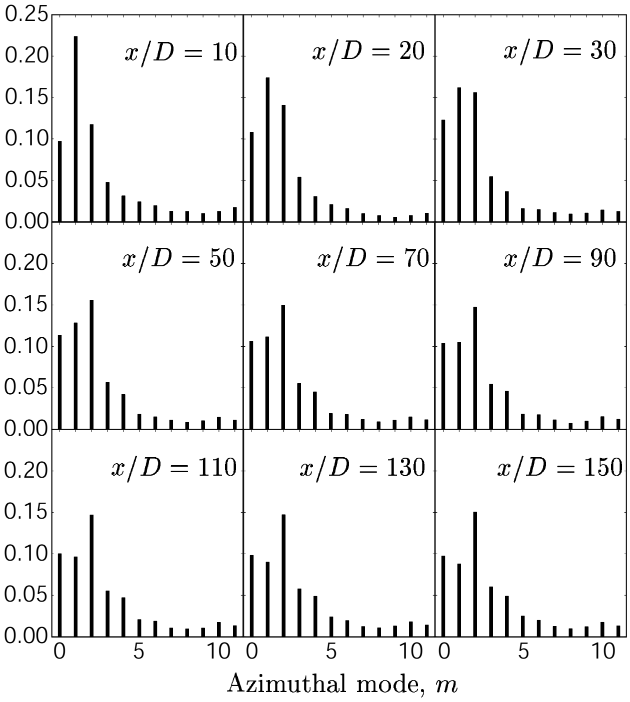

Fig. 11 Energy distribution with azimuthal mode number for the lowest order POD mode of the axisymmetric disk wake. From Johansson (2002).

stant until about $x/D = 50 - 70$. The implications of this for attempts to study axisymmetric wakes are profound, since most attempts seldom measure much beyond this point due to the extremely low turbulence intensities and limited wind tunnel lengths.

The eigenspectra as functions of m and f. Experimentally f is the frequency (or temporal variation) observed by the measuring apparatus. For the wake where $u'/U < 10\%$, Taylor's hypothesis is certainly valid, at least for all but the very lowest frequencies, so we can examine how these spatial decompositions evolve downstream.

Figure 12 shows three-dimensional plots of the first eigenspectrum, $\lambda^{(1)}(m, f, x)$, for the disk wake at $x/D = 30, 50, 90$ and 150. The most striking feature is the clear separation of the frequency content of the various modes. Only mode-1 has a peak at a non-zero frequency. The other eigenspectra (of which mode-2 is predominant) all resemble the usual broadband one-dimensional spectra of turbulence which peak at zero frequency (usually due to aliasing from the unresolved directions). The eigenspectra have not been normalized, so their heights decay downstream as the wake itself decays. But even from just these four plots it is obvious that mode-1 dies more quickly than the other modes, and especially mode-2. In fact, the reason for the behaviour of the normalized azimuthal mode number plots above (Fig. 11) is clearly not that mode-2 is increasing its contribution, but that mode-1 is fading more rapidly.

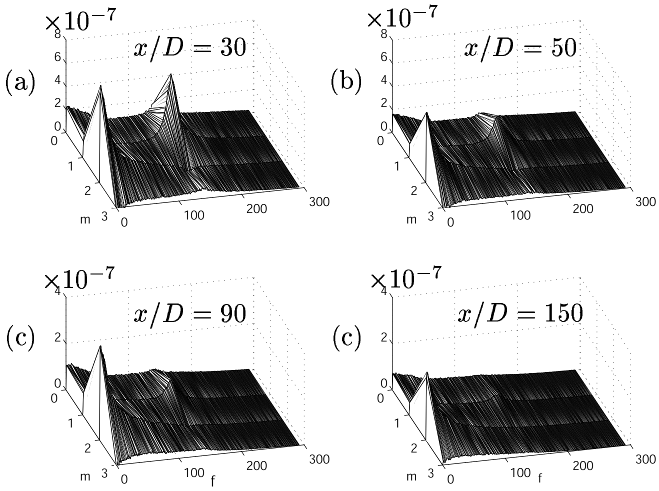

Fig. 12 Eigenspectra of first POD mode as function of frequency, azimuthal mode number and downstream distance, $\lambda^{(1)}(f, m, x)$, for the the disk wake at $x/D = 30$, 50, 90 and 150 diameters downstream. From Johansson (2002).

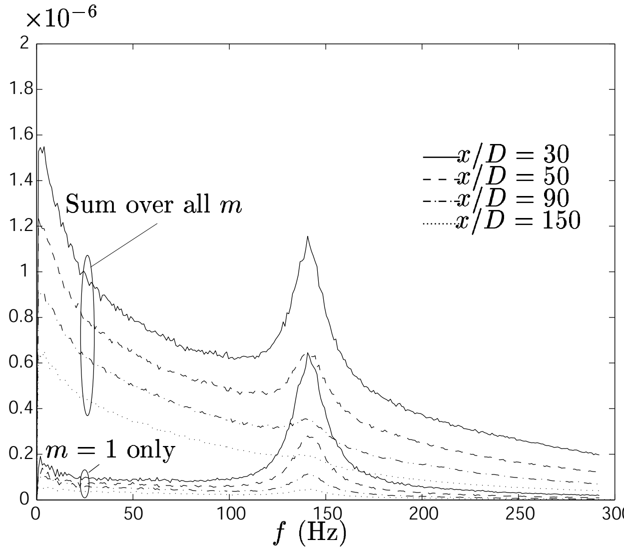

Fig. 13 Total energy (all azimuthal modes modes) and mode-1 only as a function of frequency at $x/D = 30$, 50, 90 and 150. From Johansson (2002).

Figure 13 shows plots of the total energy and mode-1 alone as a function of frequency for the same downstream positions. Most striking is that the peak frequency of the band which contains most of the energy for mode-1 does not evolve downstream, but is fixed. Moreover its contribution to the total energy is clearly diminishing downstream, as noted above. Thus the primary contribution of mode-1 clearly does not scale in local shear layer variables, but is instead determined only by the Strouhal number of the wake generator itself. It seems apparent that the primary contribution to mode-1 has been convected in from the near wake, and is virtually independent of the local shear layer of

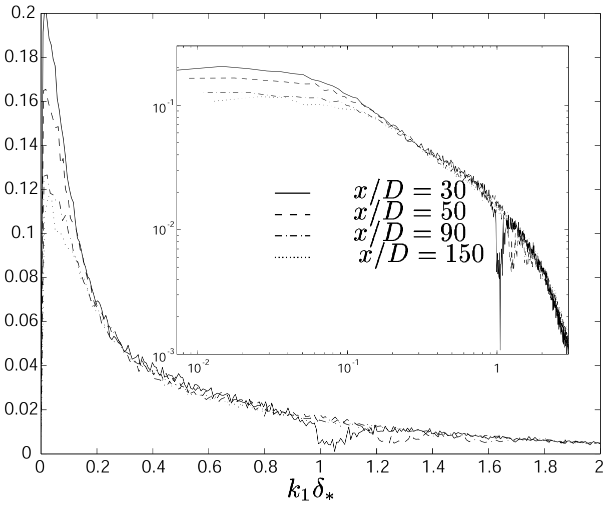

Fig. 14 Azimuthal mode-2 only at x/D = 30, 50, 90 and 150 normalized by the energy remaining AFTER the energy from azimuthal mode-1 is removed. These data have been plotted as wavenumber data using Taylor's hypothesis. Note especially the apparent suppression of mode-2 at the peak frequency of mode-1 for x/D = 30. From Johansson (2002).

the wake.

By contrast, the behaviour of mode-2 is quite different. Figure 14 shows mode-2 normalized by the energy remaining *after* the energy from mode-1 is removed. These data have been plotted as wavenumber spectra using Taylor's frozen field hypothesis. Note first of all the remarkable 'notch' in mode-2 (all the way to zero!) for the position closest to the disk at exactly the frequency where mode-1 is dominant. Clearly mode-1 is suppressing the development of mode-2 at the dominant frequency. As the wake develops downstream, this notch fills in, and except for the very lowest wavenumbers (for which Taylor's hypothesis is of doubtful validity), these data collapse wonderfully in shear layer variables. Thus, once the contribution of mode-1 has been removed, the rest of the turbulence behaves exactly as might be expected from an equilibrium similarity wake. This is certainly not the case if mode-1 is not removed, which explains the frustrations of many authors in trying to explain their measurements for this flow.

One additional observation can be made. There is a very interesting problem presented by the lack of collapse of the spectra for mode-2 at very low wavenumbers (or more properly, very low frequencies). These very low frequencies (or perhaps large scales) clearly satisfy Townsend's idea of the large eddies. They contain about 5 - 10 % of the energy and do not appear to interact with the main motion. Interestingly, if these data are NOT normalized as wavenumbers, but simply by the energy present at all mode numbers with mode-1 removed, they collapse without any scaling of the frequency axis at all. So what is their role, if any? This is not at all clear as of this writing. One possibility is that they simply slowly warp and twist the mean flow. If so this could account for the remarkably high *local* turbulence intensity for this flow for which at the centerline $u'/(U_\infty - U) \approx 130\%$! In effect, it appears the mean profile is simply being moved around by this very large and slow modulation. There is some evidence for this in the azimuthally averaged instantaneous DNS profiles of Johansson et al. (2002).

Several things are clear from the above. First, the near wake structure (mostly azimuthal mode-1) can persist far downstream and considerably complicate a simple scaling analysis. Second, it appears that this transient structure can interact with the structures that eventually dominate the far field. In particular it appears to suppress them early, and possibly modulate them later. If so, then it is reasonable to guess that different upstream conditions might interact in different ways. Perhaps it is this interaction that sets the stage for the final asymptotic state.

We have raised more questions than we have answered. Clearly we will not know the answers until more detailed information is available and dynamical models can be constructed. LES (especially together with the POD) can play an important role in this development, especially given the low Reynolds number limitations of DNS and the difficulty of experiments.

Summary and Conclusions

The evidence for the role of initial (or upstream) conditions has been briefly reviewed and shown to be consistent with a properly done equilibrium similarity analysis. Moreover, from the same analysis it was possible to show why the traditional view of asymptotic independence has arisen. In particular, the mean velocity profile and properly scaled Reynolds stress profile for the axisymmetric wake were shown to be independent of all upstream effects, even though these effects could dominate the growth rate and scale parameters and other moment profiles. Similar considerations apply to many other canonical flows.

It was further noted that single point (RANS) gradient-based turbulence models appear to be fundamentally flawed in that the effects of the initial conditions appear in the coefficients. Thus even if the models have the right functional dependence, the asymptotic state is determined by the particular set of parameters used, and no universal set is possible. LES, by contrast, appears to retain the necessary physics, whatever it may be.

Finally, some recent POD studies were reviewed to illustrate how upstream conditions might interact with the developing turbulence, and permanently modify it. It remains to be seen if dynamical systems and stability models can account for this behavior. Regardless, it appears that LES can be used as an important tool for these studies (perhaps together with the POD) IF careful attention is paid to the importance and sensitivity to initial and boundary conditions.

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